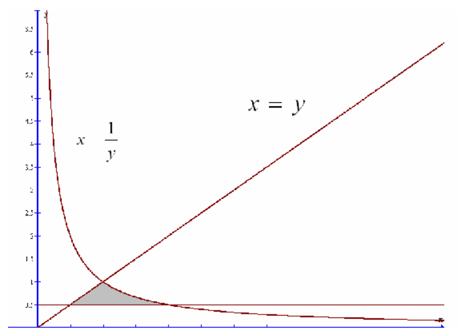
Homework #10 Solutions

Sinan Ozdemir

November 21, 2012

1. #14 on page 321

   if we are to compute the area between the curves $y = \frac{1}{2}$, $xy = 1$, and $y = x$ in terms of y then it helps to first visualize our region.



We know the integral that we must compute is of the form: $\int_c^d f(x) - g(x)\mathrm{d}x$ where c is the lower y bound, d is the upper y bound, g is the function bounding the right side of the region, and f is the function bounding the left of the region. It is easy to see that the lower y bound of the region is $\frac{1}{2}$. the upper bound, however, must be found by finding where the functions $xy = 1$ and $y = x$ intersect. We can find this by plugging in $y = x$ into $xy = 1$ and obtaining:

$$xy = 1 \text{ (now let } y = x)$$
$$y^2 = 1$$
$$y = d = 1$$

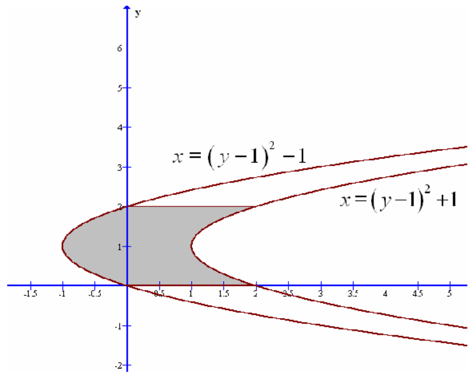
so our upper bounds are $c = \frac{1}{2}$ and $d = 1$. As far as our functions, it's clear to see that the function on the right is $y = \frac{1}{x}$ and $y = x$ on the right. Now all we have to do is write each function in terms of x. This is simple and we obtain the integral:

$$\int_{\frac{1}{2}}^{1} \frac{1}{y} - y\,\mathrm{d}y \text{ and when we integrate we get:}$$
$$ln(y) - \frac{y^2}{2}\Big|_{\frac{1}{2}}^{1} \text{ and when we plug in we get:}$$
$$(ln(1) - \tfrac{1}{2}) - (ln(\tfrac{1}{2}) - \tfrac{1}{8})$$

$$-ln(\tfrac{1}{2}) - \tfrac{3}{8} = ln(2) - \tfrac{3}{8} \text{ is the area of the region.}$$

2. #16 on page 321

   We are given the functions $x = (y-1)^2 - 1$ and $x = (y-1)^2 + 1$. When graphed, we have:



We also have the functions already in terms of y, which is all we need. All we need to do is solve the integral:

$$\int_0^2 (y-1)^2 + 1 - (y-1)^2 - 1 \mathrm{d}x = \int_0^2 2 \mathrm{d}x = 4$$

So we have our final answer: the area is 4.

3. #26 on page 322

   we use the formula in the book to get

$$\frac{1}{1-(-1)} \int_{-1}^{1} sin(\pi t) \mathrm{d}x$$
$$\frac{1}{2} \frac{-cos(\pi t)}{\pi} |_{-1}^{1}$$
$$\frac{1}{2}(0) = 0$$

   So the average value is 0.

   This makes sense because cosine is an odd function and the interval $[-1, 1]$ is centred around the origin.