Solutions

1. $\boxed{9}$.

2. The area cut out of the pizza is $100 \arccos\left(\frac{4}{5}\right) - 48$ square inches, so the answer is $\boxed{3840}$.

3. The answer is $4 - \sqrt{3} - \frac{2\pi}{3}$. Thus, $4 \times 3 \times 2 \times 3 = \boxed{72}$.

4. $\boxed{14}$.

5. $\boxed{40}$.

6. We conclude that $a = \frac{3\sqrt{3}}{2}$, so the answer is $\boxed{332}$.

7. The desired ratio is $4 : 25 : 21$, so the answer is $4 + 25 + 21 = \boxed{50}$.

8. The locus of all possible locations of P is an arc of a circle, so let's define ω as the circumcircle of $\triangle PBC$. Specifically, a $135°$ angle inscribed in a circle intercepts a $270°$ arc, meaning the locus of all P is a $90°$ arc. Also, note that $\triangle ABC$ is isosceles, with $AB = AC$. Because the measure of $\widehat{BPC}$ plus the measure of $\angle A$ is $90° + 90° = 180°$ and because $AB = AC$, $\overline{AB}$ and $\overline{AC}$ are tangent to ω at B and C, respectively. Reflecting A over line $\overleftrightarrow{BC}$ gives O, the center of ω. Because $ABOC$ is a square, the radius of ω equals AB and AC. Letting r denote the radius, we have $AB = AC = OB = OC = OP = r$. If $\triangle PAC$ is isosceles, then either $PA = PC$ or $CP = CA$.

   In the first case, P lies on the perpendicular bisector of $\overline{AC}$, meaning P lies $\frac{r}{2}$ away from $\overline{OC}$. Suppose D is the point on $\overline{OC}$ closest to P. $\triangle DPO$ is a right triangle with hypotenuse r and leg $\frac{r}{2}$, so the longer leg is $OD = \frac{r\sqrt{3}}{2}$ by the Pythagorean Theorem. Then, letting M be the midpoint of $\overline{AC}$, $PM = DC = r\left(\frac{2-\sqrt{3}}{2}\right)$. Therefore, $\tan \angle PAC = \frac{r\left(2-\sqrt{3}\right)/2}{r/2} = 2 - \sqrt{3}$.

   Now, notice that forcing $PA = PC$ in the first case forces $PB = PO = r$. Therefore, $\triangle PAB$ and $\triangle PAC$ are simultaneously isosceles, and simply interchanging B with C gives us our second case. Doing this makes the new $\angle PAC$ the complement of the old $\angle PAC$ that is now $\angle PAB$. Since $\tan \angle PAB$ is now $2 - \sqrt{3}$, $\tan \angle PAC = \frac{1}{2-\sqrt{3}} = 2 + \sqrt{3}$. Of these two options, $2 + \sqrt{3}$ is the larger value of $\tan \angle PAC$, so we use $s = 2$, $t = 1$, and $u = 3$ to obtain the answer $\boxed{213}$.

9. Quadrilateral $ABEC$ is cyclic, so $m\angle ABE = 180° - m\angle ACE$. Since $m\angle ABE$ also equals $180° - m\angle ABP$, we see that $\angle ABP \cong \angle ACE$ and thus $\triangle ABP \sim \triangle ACQ$ by angle-angle similarity. This means that $\angle BAX \cong \angle CAY$, so $BX = CY$, and therefore $BXCY$ is an isosceles trapezoid. Since the diagonals of any isosceles trapezoid are equal, $XY = BC = \boxed{69}$.

10. Let N be the intersection of the perpendicular bisector of $\overline{QR}$ with $\overline{PQ}$. Then, $NQ = NR$ and $m\angle Q = m\angle QRN$, so $m\angle PRN = 10°$. Because $\overleftrightarrow{NM}$ is the perpendicular bisector of $\overline{QR}$, $m\angle NMQ = 90°$, and because $m\angle PMQ = 100°$, $m\angle PMN = 10°$. Observe that $\angle PRN \cong \angle PMN$, so $PRMN$ is cyclic. Then, $m\angle P = 180° - m\angle RMN = 90°$, so $m\angle Q + m\angle R = 90°$. Because $m\angle R = m\angle Q + 10°$, we obtain $m\angle Q = \boxed{40°}$.