Lecture 3:.

1.7 The Isoperimetric inequality. We stated the isoperimetric inequality without proof.

2.1 Regular Surface.

Def $S \subset \mathbf{R}^3$ is a *regular surface*, if for each $p \in S$ there is a neighborhood V of p in $\mathbf{R}^3$ and a map $\mathbf{x} : U \to V \cap S$ of an open set U onto $V \cap S$ such that

1. $\mathbf{x}$ is differentiable.
2. $\mathbf{x}$ is a homeomorphism, i.e. continuous and one-to-one with continuous inverse.
3. For each $q \in U$ the differential $d\mathbf{x}_q$ is one-to-one.

Condition 3 is the regularity condition. The differential is the linear part of the map, whose definition we will come back to in the next lecture. For now this just means that

$$\mathbf{x}_u = \frac{\partial \mathbf{x}}{\partial u} = \Big(\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u}\Big), \quad \text{and} \quad \mathbf{x}_v = \frac{\partial \mathbf{x}}{\partial v} = \Big(\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v}\Big)$$

are linearly independent, which is equivalent to $\mathbf{x}_u \wedge \mathbf{x}_v \neq 0$. Condition 3 will guarantee the existence of a *tangent plane* at all points of S see Problem 2.2.3 in the book where its not satisfied.

Condition 2 will prevent the surface from self intersecting see problem 2.2.10 in the book where its not satisfied.

For an example of a parametrization see Example 2.2.1 in the book.

Appendix A Continuity. We gave the definition of continuity first for a map of an open set and then for a map of any set.

Appendix B Differential, 2.2 Regular Surface.

We started with going the definition of the differential of a map from in Appendix B to Chapter 2:

We did Example 9 of a curve going through a point in any direction.

We also did the definition 1 of the differential of a map from $\mathbf{R}^n$ to $\mathbf{R}^m$.

We also proved Proposition 7, that the differential is independent of the particular curve going through the point with the same tangent vector. In doing so we also gave the formula for the differential in terms of matrix multiplication with the Jacobian.