Math 211 Practice Midterm 2, Fall 11, Lindblad.

1. (a) Use Lagrange's method to find the point on the plane $x+2y+3z=1$ closest to the point $(2,1,0)$.

(b) Find the critical points of $f(x,y)=x^2+y^2+x+y+1$ and determine if they are local max min or saddle points.

2. (a) Let $\mathbf{F}=\mathbf{i}+\mathbf{j}$ and let $\mathbf{c}_1$ be the curve $\mathbf{c}_1=\cos t\,\mathbf{i}+2\sin t\,\mathbf{j}+t\,\mathbf{k}$, $0\le t\le\pi/2$. Find the line integral $\displaystyle\int_{\mathbf{c}_1}\mathbf{F}\cdot d\mathbf{s}$.

(b) Let $\mathbf{F}=\mathbf{i}+\mathbf{j}$ and let $\mathbf{c}_2$ be the straight line segment from $(1,0,0)$ to $(0,2,\pi/2)$. Find the line integral $\displaystyle\int_{\mathbf{c}_2}\mathbf{F}\cdot d\mathbf{s}$.

3. Find $\displaystyle\iint_R (x^2+y^2)xy\,dxdy$, where R is the region

$R=\{(x,y);\ 1\le y^2-x^2\le 2,\ \ 2\le xy\le 3\}$

by making the change of variables $u=y^2-x^2$ and $v=xy$.

4. Let R be the 3-dimensional region $R=\{x^2/4+y+z^2/4\le 1,\ y\ge 0\}$. Let S be the surface of R with the normal oriented outwards. Note that S has two parts $\{x^2/4+y+z^2/4=1,\ y\ge 0\}$ and $\{y=0,\ x^2/4+z^2/4\le 1\}$.

a) Find the area of S.

b) Find the flux of $\mathbf{F}=x\mathbf{i}-y\mathbf{j}+\mathbf{k}$ through S; $\displaystyle\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$.

5. A parametrization of the solid torus E is given

$$x=2\cos\theta+r\cos\theta\,\cos\phi,\qquad y=2\sin\theta+r\sin\theta\cos\phi,\qquad z=r\sin\phi,$$

where $0\le r\le 1$, $0\le\theta\le 2\pi$ and $0\le\phi\le 2\pi$. A parametrization for the surface of the torus T is obtained from this by putting $r=1$.

(a) Calculate the surface area element dA in terms of $d\theta d\phi$ and use it to calculate the area of the surface of the torus T.

(b) Calculate the volume element dV in terms of $drd\theta d\phi$ and use it to calculate the volume of the solid torus E.