Math 211 Practice Midterm 1, Fall 2011, Lindblad.

1. Let ϕ be a function such that $\frac{\partial \phi}{\partial x}(0,0,0) = 2$, $\frac{\partial \phi}{\partial y}(0,0,0) = 3$ and $\frac{\partial \phi}{\partial z}(0,0,0) = 4$.

(a) Let $w(t) = \phi\big(\mathbf{c}(t)\big)$, where $\mathbf{c}(t) = t\,\mathbf{i} + t^2\mathbf{j} + 3t\mathbf{k}$ is a curve. Find $\frac{dw}{dt}(0)$!

(b) In which direction is the rate of increase of ϕ largest at the point $(0,0,0)$?

(c) Let $\mathbf{F} = \mathbf{grad}\ \phi$. Find $\mathbf{curl}\,\mathbf{F}$.

2. Let $F(x,y,z) = 8x^2 + \sin^2(\pi x) + y^2 + 2z^2$ and consider the surface S given by $F(x,y,z) = 12$ and point $p = (1,2,0)$ on the surface.
(a) Find the equation for the tangent plane at p.
(b) Show that it is possible to solve for x on S as a function of (y,z) close to p.
(c) Let $x = f(y,z)$ be the function in (b). Find $Df(2,0)$.

3. Let $\mathbf{G} = -y\mathbf{i} + x\mathbf{j}$ be a vector field.

(a) Sketch the vector field $\mathbf{G}$ at the points $(1,0)$, $(0,1)$, $(-1,0)$ and $(0,-1)$ and sketch the flow line passing through $(1,0)$.

(b) Find the flow lines analytically.

4. Let $\mathbf{F}(x,y,z) = (y^2 + x)\,\mathbf{i} - (x^2 - y)\,\mathbf{j} + z\mathbf{k}$.

(a) Find $\mathbf{curl}\,\mathbf{F}$.

(b) Find $\operatorname{div}\mathbf{F}$.

(c) Find the derivative matrix $\mathbf{DF}$ (i.e. the matrix of partial derivatives).

5. Let $f(x,y) = x\cos(x+y)$

(a) Calculate the second order Taylor polynomial of f about the point $(1,-1)$.

(b) Use your answer to (a) to write down an estimate for $f(1.1,-0.8)$.

(c) Use the linear approximation to find an estimate for $f(1.1,-0.8)$.