April 1, 2009

Name ..................
Section/ Name of your TA ..................

Midterm Exam 2 100 pts.
Math 201 Ver ****

- There are 6 pages in the exam including this page.
- Write all your answers clearly. You have to show work to get points for your answers.
- Read all the questions carefully and make sure you answer all the parts.
- You can write on both sides of the paper. Indicate that the answer follows on the back of the page.
- Use of Calculators is not allowed during the exam.

(1) ...... /22
(2) ...... /20
(3) ...... /22
(4) ...... /36
Total ...... /100
(1) 22 pts. Let \( A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 0 & 0 \\ 4 & 1 & 0 \end{bmatrix} \).

(a) Find the determinant of \( A \). Show work.

(b) Find the classical adjoint of \( A \). Show work.

(c) What is the inverse of \( A \)? Show work.
(2) 20 pts. Let \( P_1 \) be the set of polynomials with degree less than or equal to 1, that is, \( P_1 = \{f(t) = a_0 + a_1 t : a_0, a_1 \in \mathbb{R}\} \). Then both \( B_1 = \{1, t\} \) and \( B_2 = \{1, t - 1\} \) are bases of \( P_1 \).

(a) What is the \( S \) matrix that transforms a vector in \( B_2 \)-coordinates into \( B_1 \)-coordinates? Show work.

(b) Let \( T : P_1 \to P_1 \) be the transformation defined as \( T(a_0 + a_1 t) = a_0 + a_1 (2t - 1) \). Find the matrix \( B \) of the transformation \( T \) with respect to the basis \( B_2 \). Show work.

(c) Let \( f \in P_1 \) be written as \( [f]_{B_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \) in \( B_2 \)-coordinates. Let \( T \) be as in part (b), then find \( [T(f)]_{B_1} \), that is, find \( T(f) \) in \( B_1 \)-coordinates. Show work.
(3) 22 pts. (a) Let \( \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \) and \( \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ -1 \end{bmatrix} \) be two different sets in \( \mathbb{R}^4 \) spanning the same subspace. Which of these two sets are orthogonal? Show work.

(b) Let \( \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \) be a basis of a subspace \( V \) of \( \mathbb{R}^4 \). Find an orthonormal basis of \( V \). Show work.
(4) 36 pts. These are all short answer questions. Explain your answer. Each of these problems is worth 12 points.

(a) Let $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 6$. Find $\det \begin{bmatrix} a - d & b - e & c - f \\ g & h & i \\ 4d & 4e & 4f \end{bmatrix}$. Explain your answer.

(b) The space of polynomials of degree less than or equal to 1, $\mathcal{P}_1$, is isomorphic to the space of complex numbers $\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$. State true or false. Give reasons.
(c) Find the volume of the parallelepiped which has the vectors \[
\begin{bmatrix}
2 \\
-1 \\
1 
\end{bmatrix},
\begin{bmatrix}
1 \\
2 \\
1 
\end{bmatrix} \text{ and } \begin{bmatrix}
0 \\
2 \\
0 
\end{bmatrix}
\] as three edges. Show work and explain your answer.