Midterm Exam # 2
Time: 50 minutes

No books, notes, calculators. Please explain carefully all steps leading to your solutions, or risk losing credit.

Problem 1: (6 points=2+1+1+2) Consider the plane $E$ in $\mathbb{R}^3$ with equation $x_1 + 2x_2 + x_3 = 0$, and let $p$ denote the orthogonal projection onto $E$.

1. If $(u_1, u_2)$ is an orthonormal basis of $E$, write the formula for $p(v)$ in terms of $u_1$ and $u_2$ (where $v$ is any vector in $\mathbb{R}^3$).

2. Find a basis of $E$.

3. Find an orthonormal basis of $E$.

4. Find the matrix for $p$ in the standard basis of $\mathbb{R}^3$. 
**Problem 2:** (9 points=3+2+1+1+2) Consider the linear space $P$ of polynomials with real coefficients.

1. Are the following subsets of $P$ linear subspaces? Explain why.
   - the set $E_0$ of polynomials $p$ such that $p(0) = 0$
   - the set $E_1$ of polynomials $p$ such that $p(1) = 1$
   - the set $P_2$ of polynomials of degree 2 or less
   
   Consider the linear map $f$ from $P_2$ to $P_2$ defined by $f(p(x)) = p''(x) + 3p'(x)$.

2. Find the kernel of $f$. Is $f$ an isomorphism?

3. Find the matrix for $f$ in the standard basis $(1, x, x^2)$ of $P_2$.

4. Prove that the vectors $p_1 = 2 + x$, $p_2 = 3$, $p_3 = 1 + 2x + 3x^2$ are linearly independent.

5. Find the matrix for $f$ in the basis $(p_1, p_2, p_3)$. 
**Problem 3:** (5 points=1+2+2)
Consider the matrix:

\[ M = \begin{pmatrix}
1 & 0 & 1 \\
0 & 1 & 1 \\
1 & 2 & 1
\end{pmatrix} \]

and let \( v_1, v_2, v_3 \) denote its column vectors.

1. **Prove that** \( v_1, v_2, v_3 \) **are linearly independant.**

2. **Perform the Gram-Schmidt process on** \( \{v_1, v_2, v_3\} \).

3. **Write the QR-factorization of** \( M \).