1. (25 points) Find the characteristic polynomial and all eigenvalues and eigenvectors of the matrix:

\[
A = \begin{pmatrix}
1 & 1 & 0 \\
0 & 1 & 1 \\
0 & 0 & 2
\end{pmatrix}.
\]
2. (25 points) Find the least squares line best fitting the following data:

(0, 1), (1, 2), and (-1, -3).
3. Answer the following questions true or false (5 points each):

(i) For any $n \times n$ matrix $A$, $\det(A) = \det(\text{rref}(A))$.

(ii) $\det\begin{pmatrix} 2 & 1 & 0 & 0 & 1 \\ 0 & 3 & 0 & 0 & 2 \\ 5 & 3 & 0 & 1 & 9 \\ 7 & 4 & 0 & 0 & 4 \\ 9 & 5 & 3 & 4 & 1 \end{pmatrix} = 3$.

(iii) For any $m \times n$ matrix $A$, $\ker(A) = \ker(A^T A)$. 

4. Answer the following (5 points each):

(i) Give an example of an orthogonal $3 \times 3$ matrix other than the identity.

(ii) Is it possible to find an example of a skew-symmetric invertible $3 \times 3$ matrix? Explain.
5. (25 points) Consider the vector space of continuous functions on $[0, 1]$ with inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x)dx .$$

Find the orthogonal projection of the function $h(x) = x^2$ onto the subspace spanned by $\{1, x\}$. 
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