LINEAR ALGEBRA (MATH 110.201)

MIDTERM I - 26 FEBRUARY 2016

Name: ____

Section number/TA: ________________________________

Instructions:

(1) Do not open this packet until instructed to do so.
(2) This midterm should be completed in **50 minutes**.
(3) Notes, the textbook, and digital devices **are not permitted**.
(4) Discussion or collaboration is **not permitted**.
(5) All solutions must be written on the pages of this booklet.
(6) Justify your answers, and write clearly; **points will be subtracted otherwise**.
(7) Once you submit your exam, you will not be allowed to modify it.
(8) By submitting this exam, you are agreeing to the above terms. Cheating or refusing to stop writing when time is called may result in an automatic failure.

Exercise	Points	Your score
1	12	
2	12	
3	12	
4	12	
5	16	
6	12	
7	12	
8	12	
Total	100	

Exercise 1. *Consider the following system of equations, where λ is a real number:*

$$\begin{aligned} 1W + \ 2X + \ \ 3Y + \ 4Z &= 1 \\ 5W + \ 6X + \ \ 7Y + \ 8Z &= \lambda \\ 9W + 10X + 11Y + 12Z &= \lambda^2 \end{aligned}$$

(1) *(4 points) Write down the augmented matrix of this system.*
(2) *(8 points) For which real numbers λ does this system have at least one solution? When it has at least one solution, describe all solutions.*

Solution:

Exercise 2. *Consider the following vectors in* $\mathbb{R}^3$*:*

$$\vec{x} = \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

(1) *(4 points) Calculate* $||\vec{x}||$ *and* $||\vec{y}||$.
(2) *(4 points) Calculate the distance between* $\vec{x}$ *and* $\vec{y}$.
(3) *(4 points) Are* $\vec{x}$ *and* $\vec{y}$ *perpendicular to each other?*

Solution:

Exercise 3. *(12 points) Find all vectors in $\mathbb{R}^3$ which are simultaneously perpendicular to both of the following vectors:*

$$\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \textit{and} \qquad \vec{w} = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$$

Solution:

Exercise 4. *Decide whether the following functions are linear transformations. If they are linear transformations, write down their matrices. If not, justify why not.*

(1) *(6 points)* $T : \mathbb{R}^2 \to \mathbb{R}^2$ *given by*

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x+1 \\ 3y-1 \end{bmatrix}$$

(2) *(6 points)* $T : \mathbb{R}^2 \to \mathbb{R}^3$ *given by*

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x-3y \\ 7y+2x \\ 3x-y \end{bmatrix}$$

Solution:

Exercise 5. *Consider* $\mathrm{Rot}_{\pi/4} : \mathbb{R}^2 \to \mathbb{R}^2$ *counter-clockwise rotation by* 45 *degrees, and* $\mathrm{Ref}_{\mathscr{L}}$ *reflection over the line* $\mathscr{L}$ *passing through the origin and* $\left[\begin{smallmatrix} 1 \\ 2 \end{smallmatrix}\right]$.

(1) *(8 points) Write down the matrices of* $\mathrm{Rot}_{\pi/4}$ *and* $\mathrm{Ref}_{\mathscr{L}}$.
(2) *(4 points) Write down the matrix of* $\mathrm{Rot}_{\pi/4} \circ \mathrm{Ref}_{\mathscr{L}}$.
(3) *(4 points) Write down the matrix of* $\mathrm{Ref}_{\mathscr{L}} \circ \mathrm{Rot}_{\pi/4}$.

Solution:

Exercise 6. *Consider the following matrix:*

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(1) *(8 points) Find* **two** *different* 3×2 *matrices* C *with the property that* $AC = I_2$.

(2) *(4 points) Is it possible to find a* 3×2 *matrix* B *with the property that* $BA = I_3$*?*

Solution:

Exercise 7. *(12 points) Is the following matrix invertible?*

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

If so, write down A^{-1}.

Solution:

Exercise 8. *(12 points) Consider the matrix*

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Is the vector $\vec{v} = \left[\begin{smallmatrix} 1 \\ 1 \\ 1 \end{smallmatrix}\right]$ *in* $\mathrm{Im}(A)$*?*

Solution: