Johns Hopkins University
Math 201, Spring 2007
Name:
Section:

Midterm Exam # 1
Time: 50 minutes

No books, notes, calculators. Please explain carefully all steps leading to your solutions, or risk losing credit.

Problem 1: (4 points=2+2)

1. Let \( r : \mathbb{R}^2 \to \mathbb{R}^2 \) be the rotation of angle \( \theta \) around the origin. Write the matrix corresponding to \( r \). Justify it by a drawing.

2. Let \( R : \mathbb{R}^3 \to \mathbb{R}^3 \) be the rotation of angle \( \theta \) around the line spanned by \( \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \). Write the matrix corresponding to \( R \). Justify it by a drawing.
**Problem 2:** (8 points=4+2+1+1)
Consider the matrix:

\[ A = \begin{pmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ 2 & 3 & 2 \end{pmatrix} \]

1. Is \( A \) invertible? If yes, compute its inverse and check your answer by evaluating the matrix product \( AA^{-1} \).

2. Solve the linear system \( AX = Y \) for \( X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \) and \( Y = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \).

3. What is \( \text{rref}(A) \)?

4. What is the rank of \( A \)?
Problem 3: (8 points = 1+4+3)

Consider the matrix:

\[
B = \begin{pmatrix}
1 & 4 & 5 & 2 \\
2 & 1 & 3 & 0 \\
-1 & 3 & 2 & 2
\end{pmatrix}
\]

1. What are the domain and codomain of the linear transformation \( f \) defined by \( B \)?

2. Find a basis of the image \( \text{Im}(B) \). What is the rank of \( B \)?

3. Find a basis of the kernel \( \text{Ker}(B) \). What is its dimension? How could you predict this value?