Name: ______________________________ February 28, 2006

Section Number/Time ______________________

TA ______________________

MATH 201, MIDTERM #1

Directions: This is a pencil-and-paper exam. You are asked to put away all books, notes, calculators, cell phones, and other computing and/or telecommunications equipment. The last page of this booklet is blank and is intended for use as scrap paper. Additional sheets of paper are available upon request.

Grading: There are fifteen questions on the exam, each worth 4 points, for a total of 60 points.

Special Note: Many of the problems in this exam are interrelated. If the answer to one question appears to require the answer to a previous question which you have not solved, you may instead explain how this missing information would be used to solve the problem.

Problems 1-11 will examine a wide range of properties of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ -2 & 2 & 2 \\ 3 & -1 & 3 \end{bmatrix}$.

1. What is the domain of A? [In other words, what is the domain of the function $T(\vec{x}) = A\vec{x}$?]

2. What is the codomain of A?

3. What is $\text{rref}(A)$?

4. What is the rank of A?

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We are still considering various properties of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ -2 & 2 & 2 \\ 3 & -1 & 3 \end{bmatrix}$. You may wish to copy your answer from Problem #3 in the space provided.

$\text{rref}(A) = \begin{bmatrix} & & \\ & & \\ & & \\ & & \end{bmatrix}$

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5. What is the dimension of the image of A?

6. Find a basis for the image of A.

7. What is the dimension of the kernel of A?

8. Find a basis for the kernel of A.

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$\text{rref}(A) = \begin{bmatrix} & & \\ & & \\ & & \\ & & \end{bmatrix}$

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9. Is there a vector $\vec{b}$ in $\mathbb{R}^4$ so that the equation $A\vec{x} = \vec{b}$ has exactly one solution? Explain your answer.

10. Is there a vector $\vec{b}$ in $\mathbb{R}^4$ so that the equation $A\vec{x} = \vec{b}$ has no solutions? Explain your answer.

Problems 11-15 will use the set of vectors $\mathcal{B} = (\vec{e}_1, \vec{e}_2, \vec{v}) = \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \right)$ in $\mathbb{R}^3$.

11. Suppose $\mathcal{B}$ is a basis for $\mathbb{R}^3$. What does that say about the coordinate v_3?

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We continue to work with the vectors $\mathcal{B} = (\vec{e}_1, \vec{e}_2, \vec{v}) = \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \right)$,
and we continue to assume they form a basis of $\mathbb{R}^3$.

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12. If T is a linear transformation with the properties that $T(\vec{e}_1) = \vec{0}$, $T(\vec{e}_2) = \vec{0}$, and $T(\vec{v}) = \vec{v}$, what is the $\mathcal{B}$-matrix of the transformation T? Call this matrix B.

13. What is the image of T?

14. Give a geometric description of what the linear transformation T does to vectors in $\mathbb{R}^3$.

15. The linear transformation T can also be written in standard coordinates as $T(\vec{x}) = A\vec{x}$. Write a formula for the matrix A in terms of your answer to Problem 12 and the invertible matrix $S = \begin{bmatrix} 1 & 0 & v_1 \\ 0 & 1 & v_2 \\ 0 & 0 & v_3 \end{bmatrix}$.

16. [Extra Credit] Find the matrix A exactly, using your favorite method. Please write on the next page. Suggestion: We know what $T(\vec{e}_1)$ and $T(\vec{e}_2)$ are. What is $T(\vec{e}_3)$?