Name: ___________________________  February 28, 2006
Section Number/Time ___________________________  
TA ___________________________

MATH 201, MIDTERM #1

Directions: This is a pencil-and-paper exam. You are asked to put away all books, notes, calculators, cell phones, and other computing and/or telecommunications equipment. The last page of this booklet is blank and is intended for use as scrap paper. Additional sheets of paper are available upon request.

Grading: There are fifteen questions on the exam, each worth 4 points, for a total of 60 points.

Special Note: Many of the problems in this exam are interrelated. If the answer to one question appears to require the answer to a previous question which you have not solved, you may instead explain how this missing information would be used to solve the problem.

Problems 1-11 will examine a wide range of properties of the matrix \( A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ -2 & 2 & 2 \\ 3 & -1 & 3 \end{bmatrix} \).

1. What is the domain of \( A \)? [In other words, what is the domain of the function \( T(\vec{x}) = A\vec{x} \)?]

2. What is the codomain of \( A \)?

3. What is \( \text{rref}(A) \)?

4. What is the rank of \( A \)?
We are still considering various properties of the matrix \( A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ -2 & 2 & 2 \\ 3 & -1 & 3 \end{bmatrix} \), \( \text{rref}(A) = \begin{bmatrix} \end{bmatrix} \).

You may wish to copy your answer from Problem #3 in the space provided.

5. What is the dimension of the image of \( A \)?

6. Find a basis for the image of \( A \).

7. What is the dimension of the kernel of \( A \)?

8. Find a basis for the kernel of \( A \).
We are still considering various properties of the matrix \( A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ -2 & 2 & 2 \\ 3 & -1 & 3 \end{bmatrix} \), \( \text{rref}(A) = \begin{bmatrix} \end{bmatrix} \)

You may wish to copy your answer from Problem #3 in the space provided.

9. Is there a vector \( \vec{b} \) in \( \mathbb{R}^4 \) so that the equation \( A\vec{x} = \vec{b} \) has exactly one solution? Explain your answer.

10. Is there a vector \( \vec{b} \) in \( \mathbb{R}^4 \) so that the equation \( A\vec{x} = \vec{b} \) has no solutions? Explain your answer.

Problems 11-15 will use the set of vectors \( B = (\vec{e}_1, \vec{e}_2, \vec{v}) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \) in \( \mathbb{R}^3 \).

11. Suppose \( B \) is a basis for \( \mathbb{R}^3 \). What does that say about the coordinate \( v_3 \)?
We continue to work with the vectors $B = (\vec{e}_1, \vec{e}_2, \vec{v}) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ v_1 \\ v_2 \\ v_3 \end{pmatrix}$, and we continue to assume they form a basis of $\mathbb{R}^3$.

12. If $T$ is a linear transformation with the properties that $T(\vec{e}_1) = \vec{0}$, $T(\vec{e}_2) = \vec{0}$, and $T(\vec{v}) = \vec{v}$, what is the $B$-matrix of the transformation $T$? Call this matrix $B$.

13. What is the image of $T$?


15. The linear transformation $T$ can also be written in standard coordinates as $T(\vec{x}) = A\vec{x}$. Write a formula for the matrix $A$ in terms of your answer to Problem 12. The invertible matrix $S = \begin{pmatrix} 1 & 0 & v_1 \\ 0 & 1 & v_2 \\ 0 & 0 & v_3 \end{pmatrix}$.

16. [Extra Credit] Find the matrix $A$ exactly, using your favorite method. Please write on the next page. Suggestion: We know what $T(\vec{e}_1)$ and $T(\vec{e}_2)$ are. What is $T(\vec{e}_3)$?