LINEAR ALGEBRA (MATH 110.201)

MIDTERM I

Name: ____________________

Section number/TA: ____________________

Instructions:

(1) Do not open this packet until instructed to do so.
(2) This midterm should be completed in **50 minutes**.
(3) Notes, the textbook, and digital devices **are not permitted**.
(4) Discussion or collaboration is **not permitted**.
(5) All solutions must be written on the pages of this booklet.
(6) Justify your answers, and write clearly; points will be subtracted otherwise.

Exercise	Points	Your score
1	5	
2	5	
3	5	
4	5	
5	5	

Exercise 1 (5 points) Let a, b be fixed real numbers. Consider the following system of equations:

$$\begin{aligned} X + Y &= a \\ X + 2Y + Z &= b \\ X + 3Y + 2Z &= 0 \end{aligned}$$

Determine all possible values of a, b for which the above system has a solution. When the system has a solution, describe all solutions in terms of a and b.

Solution:

Solution (continued):

Exercise 2 (5 points) Suppose that $T, U : \mathbb{R}^n \to \mathbb{R}^m$ are linear transformations. Let c be a fixed real scalar. Consider the function $H : \mathbb{R}^n \to \mathbb{R}^m$ defined by $H(x) = cT(x) + U(x)$. Explain why H is a linear transformation. How does the matrix of H relate to the matrices of T and U?

Solution:

Solution (continued):

Exercise 3 (5 points) Let $w = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, and let $\mathcal{L}$ be the line in $\mathbb{R}^3$ passing through the origin and w. Let $\text{Proj}_{\mathcal{L}} : \mathbb{R}^3 \to \mathbb{R}^3$ be the function that sends a vector x to its orthogonal projection onto $\mathcal{L}$.

(1) By definition, $\text{Proj}_{\mathcal{L}}(x) = kw$ for some scalar k; express this scalar in terms of x and w.
(2) Prove, using your answer from (1), that $\text{Proj}_{\mathcal{L}}$ is a linear transformation.
(3) Write down the matrix of $\text{Proj}_{\mathcal{L}}$.

Solution:

Solution (continued):

Exercise 4 (5 points) Let a and b denote real numbers, with $a \neq 0$. Determine whether the following matrix is invertible, and write down A^{-1} if it is.

$$A = \begin{bmatrix} a & 0 & b & 0 \\ 0 & a & 0 & b \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{bmatrix}$$

Double-check that your answer is correct when $a = 1$ and $b = 1$.

Solution:

Solution (continued):

Exercise 5 (5 points) Let A be an $n \times n$ matrix. Recall that

$$\ker(A) = \{x \in \mathbb{R}^n \mid Ax = 0_n\}$$

where 0_n is the vector in $\mathbb{R}^n$ having all 0's as coordinates. Show that if $x \in \ker(A)$, then $x \in \ker(A^2)$ (where $A^2 = A \cdot A$ is the matrix product of A with itself).

Solution:

Solution (continued):