Practice Exam 1 40pts.

- There are 6 pages in the exam including this page.
- Write all your answers clearly. You have to show work to get points for your answers.
- You can write on both sides of the paper. Indicate that the answer follows on the back of the page.
- Use of Calculators is not allowed during the exam.

(1) ........ /10
(2) ........ /10
(3) ........ /8
(4) ........ /12
Total ........ /40
1(a) 7pts. Find all solutions to the given system of equations.

\[\begin{align*}
  x_1 + x_3 + x_4 &= 0 \\
  2x_1 + x_2 - x_4 &= 0 \\
  x_1 - x_2 + 2x_3 + 2x_4 &= 0 \\
  x_1 + x_2 + 3x_3 + 6x_4 &= 0
\end{align*}\]

1(b) 3pts. Are the vectors

\[\begin{bmatrix}
  1 \\
  2 \\
  1
\end{bmatrix}, \quad \begin{bmatrix}
  0 \\
  1 \\
 -1
\end{bmatrix}, \quad \begin{bmatrix}
  1 \\
  2 \\
  1
\end{bmatrix}\text{ and } \begin{bmatrix}
  -1 \\
  2 \\
  6
\end{bmatrix}\]

linearly independent? Why or why not?
2(a) 7pts. Let $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. Compute the inverse of $A$, if it exists.

2(b) 3pts. Find all possible solutions for the system

\begin{align*}
2x_1 + x_2 - x_3 &= 2 \\
x_1 + x_2 + x_3 &= 3 \\
2x_1 + 2x_2 + 3x_3 &= -1
\end{align*}
(3) 8pts. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 2 \\ 1 \\ h \end{bmatrix}$.

(a) 6pts. Find all values of $h$ such that $v = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ is in the Span\{v_1, v_2, v_3\}?

(b) 2pts. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be defined as $T(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}) = x_1 v_1 + x_2 v_2 + x_3 v_3$. For what values of $h$ is $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ is in Im$T$. 

(4) 12pts. State True or False with justification. 3pts. each for the justification.

(a) The linear transformation \( T : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) defined by \( T(v) = v + v \) is a linear transformation.

(b) If \( A, B \) and \( C \) are \( 2 \times 2 \) invertible matrices then, \( (AB)C \) is invertible.
4(c) The vectors \[
\begin{bmatrix}
4 \\
-8 \\
2
\end{bmatrix},
\begin{bmatrix}
6 \\
-12 \\
3
\end{bmatrix}
\text{ and }
\begin{bmatrix}
-2 \\
4 \\
-1
\end{bmatrix},
\] span \( \mathbb{R}^3 \).

4(d) The matrix \[
\begin{bmatrix}
1 & 1 \\
1 & -1
\end{bmatrix}
\] represents reflection about a line.