THE JOHNS HOPKINS UNIVERSITY  
Krieger School of Arts and Sciences  
SOLUTIONS TO FIRST MIDTERM EXAM - FALL 2005  
110.201 – LINEAR ALGEBRA  

Instructor: Professor Carel Faber  
Duration: 50 minutes  
October 19, 2005  

No calculators allowed  
Total = 100 points  

1. [20 points] Find the inverse of the matrix  

\[
A = \begin{bmatrix}
1 & -1 & 1 \\
3 & -2 & 8 \\
2 & -2 & 3
\end{bmatrix}.
\]

Check your answer.
\[ 2. \text{ [20 points]} \text{ Solve the linear system } \]
\[
\begin{align*}
2x_1 & + 4x_2 & - 2x_3 & - 10x_4 & + 2x_5 &= 7 \\
3x_1 & + 6x_2 & + 3x_3 & + 3x_4 & - 2x_5 &= -4 \\
x_1 & + 2x_2 & & - 2x_4 &= 1
\end{align*}
\]
Check your answer.
3. [20 points] Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation with matrix

$$A = \begin{bmatrix} 2 & -3 & 12 & 17 \\ 3 & 2 & 5 & 6 \\ 1 & 4 & -5 & -8 \end{bmatrix}$$

and let $U : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation with matrix

$$B = \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$  

It is given that $B = \text{rref}(A)$.

(a) [5 points] Determine $T \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ and $T \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$.

(b) [5 points] Determine a basis for $\ker(U)$. Show your work.

(c) [5 points] Determine a basis for $\ker(T)$. Show your work.

(d) [5 points] Determine a basis for $\text{im}(T)$. Show your work.
4. [20 points] Let \( P_2 \) be the linear space of all polynomials of degree \( \leq 2 \). It is a subspace of \( F(\mathbb{R}, \mathbb{R}) \). Consider the following elements of \( P_2 \):
\[
f_1 = 1 - 2x + x^2, \quad f_2 = 2 - 3x + 5x^2, \quad f_3 = x - 2x^2, \quad f_4 = -x^2.
\]
(a) [5 points] Prove that \( f_1, f_2, f_3, f_4 \) are linearly dependent elements of \( P_2 \).
(b) [5 points] Prove that \( f_1, f_2, f_3, f_4 \) span \( P_2 \).
(c) [5 points] Prove that \( \mathcal{B} = (f_1, f_3, f_4) \) is a basis of \( P_2 \).
(d) [5 points] Find the \( \mathcal{B} \)-coordinate vector of \( f_2 \).
5. [20 points] Here we consider linear transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$.

(a) [4 points] Let $S$ be the reflection in the line $y = -x$. Determine the standard matrix of $S$.

(b) [4 points] Let $T$ be the reflection in the line $y = x\sqrt{3}$. Determine the standard matrix of $T$.

(Note that the angle between $\vec{a} = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ equals $\pi/3$.)

(c) [4 points] Determine the standard matrix of the composite transformation $ST$.

(d) [4 points] Prove that $ST$ is a rotation and find the angle of rotation (in the counterclockwise direction).

(e) [4 points] Is $TS$ the inverse of $ST$? Explain your answer as well as you can.