THE JOHNS HOPKINS UNIVERSITY
Krieger School of Arts and Sciences
SOLUTIONS TO FIRST MIDTERM EXAM - FALL 2005
110.201 – LINEAR ALGEBRA

Instructor: Professor Carel Faber
Duration: 50 minutes October 19, 2005

No calculators allowed **Total = 100 points**

1. [**20** points] Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 3 & -2 & 8 \\ 2 & -2 & 3 \end{bmatrix}.$$

Check your answer.

2. [**20** points] Solve the linear system

$$\left|\begin{array}{rcrcrcrcrcr} 2x_1 & + & 4x_2 & - & 2x_3 & - & 10x_4 & + & 2x_5 & = & 7 \\ 3x_1 & + & 6x_2 & + & 3x_3 & + & 3x_4 & - & 2x_5 & = & -4 \\ x_1 & + & 2x_2 & & & - & 2x_4 & & & = & 1 \end{array}\right| .$$

Check your answer.

3. [**20** points] Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation with matrix

$$A = \begin{bmatrix} 2 & -3 & 12 & 17 \\ 3 & 2 & 5 & 6 \\ 1 & 4 & -5 & -8 \end{bmatrix}$$

and let $U : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation with matrix

$$B = \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

It is given that $B = \mathrm{rref}(A)$.

(a) [**5** points] Determine $T\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ and $T\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$.

(b) [**5** points] Determine a basis for $\ker(U)$. Show your work.

(c) [**5** points] Determine a basis for $\ker(T)$. Show your work.

(d) [**5** points] Determine a basis for $\mathrm{im}(T)$. Show your work.

4. [**20** points] Let P_2 be the linear space of all polynomials of degree ≤ 2. It is a subspace of $F(\mathbb{R}, \mathbb{R})$. Consider the following elements of P_2:

$$f_1 = 1 - 2x + x^2, \qquad f_2 = 2 - 3x + 5x^2, \qquad f_3 = x - 2x^2, \qquad f_4 = -x^2.$$

(a) [**5** points] Prove that f_1, f_2, f_3, f_4 are linearly dependent elements of P_2.

(b) [**5** points] Prove that f_1, f_2, f_3, f_4 span P_2.

(c) [**5** points] Prove that $\mathcal{B} = (f_1, f_3, f_4)$ is a basis of P_2.

(d) [**5** points] Find the $\mathcal{B}$-coordinate vector of f_2.

5. [**20** points] Here we consider linear transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$.

(a) [**4** points] Let S be the reflection in the line $y = -x$. Determine the standard matrix of S.

(b) [**4** points] Let T be the reflection in the line $y = x\sqrt{3}$. Determine the standard matrix of T.

(Note that the angle between $\vec{a} = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ equals $\pi/3$.)

(c) [**4** points] Determine the standard matrix of the composite transformation ST.

(d) [**4** points] Prove that ST is a rotation and find the angle of rotation (in the counterclockwise direction).

(e) [**4** points] Is TS the inverse of ST ? Explain your answer as well as you can.