Questions for Midterm III Review from Final Spring 05

5a. [2 marks] Find the eigenvalues of $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

5b. [3 marks] Give a factorization $A = QDQ^T$ where Q has orthonormal columns and D is a diagonal matrix.

5d. [1 marks] Is A a positive definite matrix? Why? Give the quadratic form $q(x, y)$ associated to A.

6a. [3 marks] If possible, find an invertible matrix M such that

$$M^{-1} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix}.$$

If it is not possible, state why M cannot exist.

6b. [3 marks] For what real values of c (if any) is

$$A = \begin{bmatrix} -1 & c & 2 \\ c & -4 & -3 \\ 2 & -3 & 4 \end{bmatrix}$$

a symmetric positive definite matrix?

6c. [4 marks] Let $A = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$. Is the quadratic form $q(x, y)$ associated to A positive definite? Find its principal axes.

7a. [6 marks] Let $A_1 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Is A_1 diagonalizable? Why? Is A_1 invertible? Why? Determine the spectral decomposition of A_1 into projection matrices.

7b. [3 marks] Let $A_2 = \begin{bmatrix} -3 & 3 \\ 1 & -1 \end{bmatrix}$. Is A_2 invertible? Why? Is A_2 diagonalizable? Why? Determine (if exists) a matrix S and a diagonal matrix D such that $S^{-1}A_2S = D$.

7c. [6 marks] Describe the linear transformation $T_{A_2} : \mathbb{R}^2 \to \mathbb{R}^2$ associated to A_2. Does such A_2 have a decomposition into projection matrices? If yes, give it.

8a. [3 marks] Find the lengths and the inner product $\underline{x} \cdot \underline{y}$ of the following complex vectors

$$\underline{x} = \begin{bmatrix} 2-4i \\ 4i \end{bmatrix}, \qquad \underline{y} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \qquad (i^2 = -1).$$

8b. [3 marks] Let $A = \begin{bmatrix} 1 & 1-i \\ 1+i & 2 \end{bmatrix}$. Let $\underline{x}_1$, $\underline{x}_2$ be two (linearly independent) eigenvectors of A. Compute $\underline{x}_1 \cdot \underline{x}_2$ and show that $\det(A) \in \mathbb{R}$.

8c. [4 marks] Prove that for any complex vector $\underline{x}$

$$\underline{x}^H A \underline{x} \in \mathbb{R}. \qquad (H = \text{Hermitian})$$