MATH 417 MIDTERM 2 THURSDAY APRIL 23

1. Consider the second order differential equation

$xu''(x) + 2u'(x) + (\lambda - x)u(x) = 0,\ 1 < x < 2,\ u(1) = u(2) = 0$.

a. (10pts) Put the equation in Sturm-Liouville form.

b. (10pts) Write out the orthogonality condition for the eigenfunctions.

c. (10pts) Show that all eigenvalues $\lambda > 0$.

2. Consider the boundary value problem
$y''(x) + 4y(x) = f(x),\ 0 < x < \pi,\ y(0) = y(\pi) = 0$.

a. (15pts) Show that a necessary condition for a solution is that

$$\int_0^\pi f(x) \sin 2x \ dx = 0$$

.

b.(15pts) Assuming this condition, find the solution by the method of eigenfunction expansion.

3a (20pts) . Find the Green's function for the problem

$$(xu')' = f(x),\ 1 < x < e,\ u(1) = 0,\ u(e) = 0\ ,$$

by direct construction from two linearly independent solutions. Hint: Find $u_1(x),\ u_2(x)$ linearly solutions of $(xu')' = 0$ with $u_1(1) = u_2(e) = 0$

3b. (20 pts) Use the Green's function to find the explicit solution for $f(x) = x$. You do not need to derive the formula for the representation of the solution in terms of the Green's function. The integration by parts formula, $\int (y \log y \ dy = \frac{1}{2}y^2 \log y - \frac{1}{4}y^2$ will be handy.
Check directly that your solution is correct.