Problem Set 7, due Tuesday March 24

Read Haberman 5.1-5.3

1. (15pts) Solve $u_{tt} = c^2 u_{xx}$, $u(x,0) = e^x$, $u_t(x,0) = \sin x$.

2. (15pts) Find $u(\frac{2}{3}, 2)$ if $u_{tt} = u_{xx}$ in (0,1) with $u(x,0) = x^2(1-x)$, $u_t(x,0) = (1-x)^2$, and , $u(0,t) = u(1,t) = 0$.
Hint: Recall that you extend the initial data to (-1,0) by odd reflection and then this data (now defined on (-1,1)) periodically of period 2.

3. (15pts) 5.3.3

4.(20pts) 5.3.4 a,b

5.(15pts) 5.3.5

6.(20pts) Find the eigenvalues and eigenfunctions for

$$(xu')' + \frac{\lambda}{x}u = 0 \text{ , on } 1 < x < b \text{ , } u(1) = u(b) = 0 \text{ .}$$

What is the orthogonality condition? (see 5.3.9) HInt: Note that all the eigenvalues are positive. To find the eigenvalues and eigenvectors, we need to use the change of variables, $x = e^t$, $w(t) = u(x)$. Verify that $w'' + \lambda w = 0$ and proceed from there.