Homework 5 (due Wed. October 17)

The Homework problems are meant to be challenging. You are encouraged to think about and discuss homework problems with your fellow students (and with Duncan Sinclair and me) but you are expected to write up solutions by yourself.

These are some exam preparation problems

p.72: 17i.ii
p.106:7
p.128:5 (hint: Use the intermediate value theorem
p137:1i,,iii,iv,vii

1. Prove by induction that $\frac{1}{1\cdot2} + \frac{1}{2\ cdot3} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$.

2. Show that $f(x) = \sin\frac{1}{x}$ is not uniformly continuous on $(0, \frac{2}{\pi}]$. Hint: Find sequences $\{x_n\}$, $\{y_n\}$ with $f(x_n) = 1$, $f(y_n) = 0$ and $|x_n - y_n| \to 0$.

3. i. prove that if $a, b \geq 0$ and $a^2 < b^2$, then $a < b$.

ii. Use part i to show that for $x \geq 0$, $\sqrt{x^2 + 6x} < x + 3$.

4. Find $\lim_{x\to\infty}(\sqrt{x^2 + 4x} - x)$. Justify.