Minimizer of a class of quartic polynomials

Fei Lu
feilu@math.jhu.edu
Last updated: 10/17/2018

Summary: seek analytical minimizer of a multivariate quartic polynomial with random coefficients and study the asymptotic behavior of the minimizer.

1 Minimization of quartic polynomial

Consider the minimization of the polynomial of $x \in \mathbb{R}^d$:

$$g(x) = \sum_{k=1}^{K} |c_k - 2x^T b_k - x^T A_k x|^2 \tag{1}$$

where $c_k \in \mathbb{C}$, $b_k \in \mathbb{C}^d$, $A_k \in \mathbb{C}^{d\times d}$ for each $k = 1, \dots, K$, and $A = A^T$.

We can assume that A, b, c are random arrays, identically distributed for different k's (in general, stationary stochastic processes indexed by k).

GOAL: The goal is to find an analytical minimizer (if possible) of g (in terms of A,b,c) and analyze its asymptotic behavior as K increases.

Numerical tests The minimizer seems to be unique (via fminunc in MATLAB) for g:

$$g(x) = \sum_{k=1}^{K} Real(c_k - 2x^T b_k - x^T A_k x)^2 + Imag(c_k - 2x^T b_k - x^T A_k x)^2$$