Real Analysis I: Midterm Exam Sample, 10/8/2025

Your name: ________________________

This is a closed-book in-class exam. **Cell phones are NOT allowed in the exam.**

1. (10 pts) True or False:

(1)_____: The set $\mathbb{Q}$ has the least-upper-bound property.

(2)_____: Let $f : D \to \mathbb{R}$ be a bounded function. Then, there exists $x \in D$ such that $f(x) = \sup_{y \in D} f(y)$.

(3)_____: Let $f, g : D \to \mathbb{R}$ be bounded functions and $f(x) \leq g(x)$ for all $x \in D$. Then, $\sup_{x \in D} f(x) \leq \inf_{x \in D} g(x)$.

(4)_____: A divergent sequence can have a convergent subsequence.

(5)_____: If both subsequences $\{x_{2n}\}_{n \geq 1}$ and $\{x_{2n-1}\}_{n \geq 1}$ converge, then $\{x_n\}$ itself converges.

2. (20 = 5+15 pts) Supremum and infimum.
(a) State the definition of the supremum of a set $A \subset \mathbb{R}$.
(b) Let $A, B \subset \mathbb{R}$ be nonempty bounded sets. Let $C = \{a + b : a \in A, b \in B\}$. Prove that

$$\sup C = \sup A + \sup B.$$

3. (20 points) Limit superior and limit inferior.
(a) State the definition of $\liminf_{n\to\infty} x_n$ for a sequence $\{x_n\}_{n=1}^{\infty}$.
(b) Let $\{x_n\}_{n=1}^{\infty}$ be a sequence in $\mathbb{R}$ such that both limit superior and limit inferior are finite. Prove that the sequence $\{x_n\}_{n=1}^{\infty}$ is bounded.

4. (25 pts) Cauchy sequence.
(a) State the definition of a Cauchy sequence in $\mathbb{R}$.
(b) Let $\{x_n\}_{n=1}^{\infty}$ be a Cauchy sequence in $\mathbb{R}$. Prove that the sequence $\{x_n\}_{n=1}^{\infty}$ is bounded.

5. (25 pts) Series.

(a) Prove that $\lim_{n\to\infty} n^{1/n} = 1$. (Hint: use ratio test on $\{\frac{n}{(1+\epsilon)^n}\}$ to show $n < (1+\epsilon)^n$ for large n.)

(b) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} n\frac{(x-2)^n}{3^n}$.

s1

s2