

MATH 616: ALGEBRAIC TOPOLOGY

EMILY RIEHL

OVERVIEW

The course lectures will be divided into three parts.

Part I: Homological algebra. Invariants of groups, rings, knots, topological spaces, and so forth are often defined by first constructing a *chain complex*, at which point the study of these invariants enters a purely algebraic domain called *homological algebra*. The first part of the course will introduce this subject with no assumed prerequisites beyond undergraduate abstract algebra and some acquaintance with the language of category theory.¹ The aim will be to introduce the derived functors Ext and Tor of the hom and tensor product and to discuss the universal coefficient theorems for homology and cohomology.

Part II: Spectral sequences. The second part of the course will introduce an important computational tool in homological algebra that is used in algebraic topology, algebraic geometry, and algebraic K-theory: namely, *spectral sequences*. There are a great many famous spectral sequences introduced with different computational aims. The specific topics to be studied in this section will depend on the interests of the participants.

Part III: Abstract homotopy theory. When the class reaches the point at which all the pages start to blur together, we'll pivot to the final part of the course, which will discuss abstract homotopy theory, particularly through the lens of Quillen's *model categories*. We will explain how a model category presents its *homotopy category* also called the *derived category* in algebraic contexts. We will reintroduce derived functors, explaining the general context for the ad-hoc constructions of Ext and Tor.

COURSE LOGISTICS

Lectures: TTh 12-1:15pm, Gilman 377

Course website: <http://www.math.jhu.edu/~eriehl/616>

Contact: eriehl@math.jhu.edu, Krieger 312

Office hours: immediately following each lecture, or by appointment.

Date: Spring 2016.

¹Fluency in category theory can be developed in parallel with this course.

References:

- *An introduction to homological algebra* by Charles Weibel.
- ...
- “Homotopy theories and model categories” by W.G. Dwyer and J. Spalinski

ASSESSMENT

Problem sets. Problem sets will be assigned periodically, roughly every 2-3 lectures, and due in each case in class one week later. Collaboration is encouraged but please write up your own solutions and acknowledge your collaborators.

Oral presentation. Sometime in late February or early March each student will be expected to give one oral in-class presentation introducing a spectral sequence of their choosing. While preparing the lecture, the students should also prepare lecture notes to be distributed to the rest of the class. Handwritten notes are fine. The aim of this assignment is as much to practice both oral and written exposition as it is to learn the new material. In particular, each student will receive ample individual feedback.

Final. In lieu of a final exam, there will be a final problem set, perhaps somewhat longer than normal, which will review topics covered over the course of the entire semester and on which each student should work individually, consulting course notes and other printed references as necessary. This problem set will be due during the first week of May. (There will be no class meetings during the last week of April, which will instead function as a “reading period”.)

Course grades. A numerical grade will be assigned based on the following formula: 50% problem sets, 25% oral presentation, 25% final problem set.²

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²At this point I feel compelled to point out that no one has even asked to see my grades from graduate school, so, in my opinion, numerical grades are not particularly important. The point of taking a graduate mathematics course is to learn something, and the structure of the course assignments are designed to facilitate that aim. That said, the register asks me to assign grades, and so I will.