Math 616: Algebraic Topology
Problem Set 3
due: March 22, 2016

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Exercise 1.

(i) What is a **graded chain complex**?
(ii) What is a **filtered chain complex**?
(iii) Given a filtered chain complex, define its **associated graded**, a graded chain complex.

Exercise 2. Given a filtration on a chain complex $C_\bullet$, define an induced filtration on:

(i) The module C_n.
(ii) The module Z_nC.
(iii) The module B_nC.

Exercise 3. Demonstrate by means of an example that if you apply the functor

$$H_n \colon \mathsf{Ch}_R \to \mathsf{Mod}_R$$

to a filtration of a chain complex $C_\bullet$ this does not necessarily directly define a filtration of H_nC.[1]

Exercise 4. Prove that the spectral sequence associated to a finitely filtered chain complex collapses. At what stage?

Exercise 5. Let $C_\bullet$ be a chain complex with $C_n = 0$ if $n < 0$. Define a filtration by

$$F_pC_d := \begin{cases} C_d & \text{if } p \geq d \\ 0 & \text{if } p < d. \end{cases}$$

Note that this filtration is not typically finite.

(i) Compute the associated graded.
(ii) Compute the E^0, E^1, and E^2 pages and prove that the spectral sequence collapses.

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[1] Instead, there is a filtration on H_nC whose pth stage is defined to be the *image* of the homology of the pth stage of the filtered chain complex.