MATH 601: ALGEBRA

EMILY RIEHL

> What is algebra? Is it a branch of mathematics, a method or a frame of mind?
>
> - Igor Schafarevich, *Basic Notions of Algebra*, §1

> The language of categories is affectionately known as *abstract nonsense*, so named by Norman Steenrod. This term is essentially accurate and not necessarily derogatory: categories refer to *nonsense* in the sense that they are all about the 'structure', and not about the 'meaning', of what they represent.
>
> -Paolo Aluffi, *Algebra: Chapter 0*, §I.3

Overview

The aim of this course will be to (re)introduce the basic language of modern algebra. In the first term, we hope to cover groups and their actions, rings and their modules, and a substantial amount of linear algebra, with these results proven in the more general setting of modules over a commutative ring when such generalizations are possible. Our presentation will adopt a somewhat categorical viewpoint, by which we mean that mathematical constructions admitting an interesting property will be characterized in this way. No prior acquaintance with category theory is assumed. The basic notions of category and functor and initial and terminal object will be defined early in the term. A more thorough account of the meaning of the phrase "universal property" will be presented in the middle once we have encountered a sufficient array of examples upon which to cut our teeth.

In fact, strictly speaking no prior acquaintance with algebra is necessary as we will present all of the relevant definitions in class. This said, our pace will be quite rapid, so it is disingenuous to suggest that this course would serve well as a first introduction to abstract algebra.

Course-level learning goals

By the end of the semester I hope that each of you has:

- learned a substantial amount of algebra and internalized some basic categorical language.
- strove to develop a clear, concise voice through writing solutions to the weekly problem sets.

Date: Fall 2017.

- developed the habit of *actively* engaging with new or familiar mathematical concepts, by asking questions, seeking comparisons with related mathematical ideas, and searching for examples or counter-examples.
- engaged in frequent mathematical discussions with many of your classmates, for instance in collaborating on homework problems or reviewing for exams.

Textbook

We will follow Paolo Aluffi's *Algebra: Chapter 0*, which, unlikely the canonical reference book (Serge Lang's *Algebra*) is designed to be *read* from cover to cover. I suggest that you do so, both to reinforce the material that we will discuss in lecture but equally because Aluffi's text is a model of clear mathematical exposition. One of the program-level learning goals for graduate students in mathematics is to learn to write beautifully, so that your papers *compel* your audience to read them. This is not easy (particularly for those of you whose native language is not English) but one way to get started is to read samples of model mathematical exposition.[1]

I hope, perhaps over-ambitiously, to cover most of chapters I-VI, with the exceptions of §I.1, §I.2, §II.10, §III.7, and §IV.6 (which is subsumed by §VI.5).

Other References:

- For more algebra: Serge Lang's *Algebra* or an alternate textbook of your choice and language preference
- For more category theory: *Category theory in context*, available here: http://www.math.jhu.edu/~eriehl/context.pdf

Course logistics

Lectures: MW 9-10:15, Bloomberg 176

Course website: http://www.math.jhu.edu/~eriehl/601

Contact: eriehl@math.jhu.edu, Krieger 312

Office hours: Tuesdays 1-3pm or by appointment

TA:. Sarah Yu, syu43@math.jhu.edu, Krieger 201, office hours: Fridays 2-3pm

What you can call me

You are welcome to address me as "Professor Riehl," "Dr. Riehl," or "Emily." I insist that my PhD students use the latter; for the rest of you, I have no strong preference. I use she/her pronouns.

Assessment

[1] My other recommendation to develop your mathematical voice is to practice: write, write, write, speak, speak, speak, LaTeX, LaTeX, LaTeX, then repeat ...

Problem sets. A problem set will be due each week in class on Wednesday, with the exception of the first Wednesday meeting and the Wednesday after the midterm. Late homework will be accepted only with an exceptionally good excuse.

Collaboration on homework is allowed and encouraged. However, each student must write up their solutions to the problems individually and in their own words, and must acknowledge their collaborators by name on their written assignments. Copying from another student or any other source is prohibited.[2]

Class participation requirement. To satisfy the class participation requirement, each student must ask one question or make one comment in class at least once before the midterm and at least once after the midterm. An acceptable question is: "would you remind us what X means."[3] I will do everything I can to help everyone satisfy the class participation requirement.

Exams. There will be an in-class midterm exam on Monday, October 23 and a final exam to be scheduled early in exam period.[4] Both exams will be designed to simulate the graduate qualifying exam in algebra,[5] though the questions will be restricted to the material that we will have covered up to that point. Both exams will be graded "pass" (full credit), "attempted" (partial credit), "absent" (no credit). The exams will be hard; in particular, it will not be necessary to solve all of the problems to pass the exam.

Course grades. A numerical grade will be assigned based on the following formula:[6]

- Any student who passes the final exam and attempts the midterm exam will receive their cumulative problem set score as their final grade.
- Any student who passes the midterm exam and attempts the final exam will receive 95% of their cumulative problem set score as their final grade.
- Any student who attempts both exams will receive 90% of their cumulative problem set score as their final grade.
- Any student who does not attempt an exam or who does not complete a substantial portion of the homework will receive an incomplete.

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[2]The policies of the Johns Hopkins Ethics Guide will apply to this course.

[3]In the not unlikely event that I don't in fact remember what X means, an acceptable comment would be the answer to this question. One of the reasons the categorical point of view appeals so strongly to me is I find it makes mathematics easier to remember.

[4]The registrar-designated time for our final exam is Thursday, December 21 from 9am-12pm. I would prefer to finish our course sooner than this.

[5]Disclaimer: I personally have nothing to do with the writing of the qualifying exam.

[6]At this point I feel compelled to point out that no one has even asked to see my grades from graduate school, so, in my opinion, numerical grades are not particularly important. The point of taking a graduate mathematics course is to learn something, and the structure of the course assignments are designed to facilitate that aim. That said, the register asks me to assign grades, and so I will.