

Math 411: Honors Algebra I
Midterm

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Conventions: In what follows \mathbb{Z} denotes the group of integers under addition, \mathbb{Z}/n denotes the cyclic group under addition modulo n , D_{2n} denotes the dihedral group of order $2n$, and S_n denotes the symmetric group of permutations of n elements.

TRUE OR FALSE

- (1 point) Indicate whether each of the following statements is true or false (circle one).
(2 points) For each true statement, give a short (one to two sentence) justification, explaining the essential reason for its correctness; for each false statement, provide either a counter-example or, if a counter-example would not make sense, a short disproof.

1. (T or F) Any surjective function between sets admits a right inverse.

2. (T or F) The set of all 3×3 matrices with real coefficients forms a group under matrix multiplication.

3. (T or F) All elements of $\mathbb{Z}/12$ have order 12.

4. (T or F) The product of two finite groups is a finite group.

5. (T or F) There exists a non-zero homomorphism $\mathbb{Z}/12 \rightarrow \mathbb{Z}$.

6. (T or F) There exists a non-zero homomorphism $\mathbb{Z} \rightarrow \mathbb{Z}/12$.

7. **(T or F)** The dihedral group D_{14} is generated by the rotations.

8. **(T or F)** Any subgroup of an abelian group is an abelian group.

9. (T or F) The kernel of any group homomorphism is isomorphic to the image of that group homomorphism.

10. (T or F) The subgroup of S_6 comprised of those permutations that “fix,” that is do not move, the 6th element¹ is a *normal* subgroup of S_6 .

¹You can check if you like (though you don't need to) that this subgroup is isomorphic to S_5 .