Math 411: Honors Algebra I
Problem Set 9
due: November 13, 2019

Emily Riehl

Exercise 1. Prove that a group homomorphism $\phi\colon G \to H$ is injective if and only if $\ker\phi = \{e\}$.

Exercise 2. How many elements of S_8 are conjugate to $(12)(345)(678)$?

Exercise 3. The **center** of a group G is the subgroup

$$Z(G) = \{g \in G \mid \forall x \in G, gx = xg\}$$

made up of all elements that commute with all other elements of g.

(i) Prove that $Z(G)$ is a subgroup.
(ii) Prove that $g \in Z(G)$ if and only if the conjugacy class of g contains a single element.
(iii) Prove that $Z(G)$ is normal in G.
(iv) Prove that any subgroup $H \subset Z(G)$ is a normal subgroup of G.

Exercise 4. Recall a group is *simple* if it has no non-trivial normal subgroups. Prove that any non-zero group homomorphism whose domain is a simple group is injective.

Exercise 5. Show that any group of order mp with p prime and $1 < m < p$ is not simple.

Exercise 6. Classify all finite simple *abelian* groups. (Hint: start by thinking about the order of the group.)

Dept. of Mathematics, Johns Hopkins Univ., 3400 N Charles St, Baltimore, MD 21218
E-mail address: `eriehl@math.jhu.edu`