Math 411: Honors Algebra I
Problem Set 8
due: November 8, 2017

Emily Riehl

Exercise 1. Find the center of D_{2n}. [Hint: the answer depends on whether n is even or odd.]

Exercise 2. Prove that the center of S_n is trivial for $n \geq 3$.

Exercise 3. If $H \subset G$ is a subgroup its conjugate subgroups are the subgroups of the form

$$gHg^{-1} = \{ghg^{-1} \mid h \in H\}$$

for some $g \in G$.

(i) Prove that gHg^{-1} is a subgroup of G.
(ii) Define a bijective group homomorphism $H \to gHg^{-1}$.
(iii) The group G acts on the set of subgroups of G by conjugation: the action of a group element $g \in G$ on a subgroup $H \subset G$ is defined by $H \mapsto gHg^{-1} \subset H$. Rephrase the condition of H being a normal subgroup in terms of the orbits of this action.

Exercise 4. Prove that S_{2n} is generated by just two permutations: the transposition (12) and $(12 \cdots n)$.

Exercise 5. Find the formula for the size of the conjugacy class of a permutation of any given cycle shape in S_n.

Exercise 6. Prove that any normal subgroup of S_4 must have order 1, 4, 12, or 24.

Dept. of Mathematics, Johns Hopkins Univ., 3400 N Charles St, Baltimore, MD 21218
E-mail address: `eriehl@math.jhu.edu`