

Math 411: Honors Algebra I

Practice Midterm

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TRUE OR FALSE

- (1 point) Indicate whether each of the following statements is true or false (circle one).
(2 points) For each true statement, give a short (one to two sentence) justification, explaining the essential reason why the assertion is correct; for each false statement, provide either a counter-example or, in the case where a counter-example would not make sense, a short disproof.

1. **(T or F)** A bijective function $f: A \rightarrow B$ has a unique inverse function.

2. **(T or F)** A finite group can have elements of infinite order.

3. **(T or F)** The symmetric group S_4 has 256 elements.

4. **(T or F)** The Klein four group is abelian.

5. (T or F) The function $- + 10: \mathbb{Z} \rightarrow \mathbb{Z}$, adding 10 to any integer, defines a group homomorphism.

6. (T or F) There exists a non-zero homomorphism $\mathbb{Z} \rightarrow S_6$.

7. (T or F) Every subgroup of \mathbb{Z} is cyclic.

8. (T or F) Let $r_1 \in D_{10}$ denote the reflection through the axis of symmetry that bisects the 1st vertex of the pentagon. The subgroup generated by r_1 is a normal subgroup of D_{10} .

9. (T or F) For any non-zero group homomorphism $\phi: G \rightarrow H$ if $g_1, g_2 \in G$ are so that $g_1 \cdot g_2 = g_2 \cdot g_1$ in G , then $\phi(g_1) \cdot \phi(g_2) = \phi(g_2) \cdot \phi(g_1)$ in H .

10. (T or F) For any non-zero group homomorphism $\phi: G \rightarrow H$ if $g_1, g_2 \in G$ are so that $\phi(g_1) \cdot \phi(g_2) = \phi(g_2) \cdot \phi(g_1)$ in H , then $g_1 \cdot g_2 = g_2 \cdot g_1$ in G .