Math 401: Introduction to Abstract Algebra
Problem Set 9[1]
due: April 15, 2019

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Read. §12, §14

Exercise 1. Find an example of a group G with a subgroup H so that

$$\{(x, y) \mid xy \in H\}$$

is not an equivalence relation on G.

Exercise 2. Find an example of a group G with a subgroup H so that

$$\{(x, y) \mid xyx^{-1}y^{-1} \in H\}$$

is not an equivalence relation on G.

Exercise 3. For $G = A_4$ work out the left and right cosets of

(i) the subgroup $H = \{e, (12)(34), (13)(24), (14)(23)\}$ and
(ii) the subgroup $K = \{e, (123), (132)\}$

Verify that

(i) the left and right cosets coincide in the first case: $gH = Hg$ for all $g \in G$
(ii) but not in the second: for some $g \in G$, $gK \neq Kg$.

Challenge (optional): can you figure out what property holds of H but not of K that explains this?

Exercise 4. Let G be a finite group and let H be a subgroup which contains precisely half of the elements of G. Show that $gH = Hg$ for every $g \in G$.[2]

Exercises. §12 | 12.6, 12.9

Exercise 5*. Convince yourself that the braid group B_3 is infinite, non-abelian, and is generated by the two braids b_1 and b_2 shown in Figure 12.4 on page 65.[3]

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[1]Problems labelled n^* are optional (fun!) challenge exercises that will not be graded.
[2]Hint: it might be helpful to think about their complements.
[3]Note to say that an infinite group is generated by two elements b_1 and b_2 means that every other element can be written as a product involving repetitions of b_1, b_2, b_1^{-1}, and b_2^{-1}.