Math 401: Introduction to Abstract Algebra
Problem Set 6
due: March 11, 2019

Emily Riehl

Read. §5, §6

Exercise 1. In this exercise, we'll investigate the order of the various elements $\{0, \ldots, n-1\}$ in the cyclic group $\mathbb{Z}/n$.

(i) Prove that the order of $m \in \mathbb{Z}/n$ is 1 if and only if $n \mid m$.
(ii) Prove that the order of $m \in \mathbb{Z}/n$ is $n/\gcd(m, n)$.

Exercise 2. Compute the order of all of the elements in $\mathbb{Z}/12$.

Exercise 3.

(i) Show that the element m generates $\mathbb{Z}/n$ if and only if $\gcd(m, n) = 1$.
(ii) Conclude that *every* non-zero element generates $\mathbb{Z}/p$ when p is a prime.

Exercises.
§5 | 5.1, 5.2, 5.7

Exercise 4. Find a subgroup of S_4 that contains exactly six elements. How many subgroups of order 6 are there in S_4?

Exercise 5. If $\alpha, \beta \in S_n$ prove that $\alpha\beta\alpha^{-1}\beta^{-1}$ always lies in the subgroup A_n.

Exercise 6. Prove that the order of an element $\sigma \in S_n$ is the least common multiple of the lengths of the cycles which appear when σ is written as a product of disjoint cyclic permutations.

Dept. of Mathematics, Johns Hopkins Univ., 3400 N Charles St, Baltimore, MD 21218
E-mail address: `eriehl@math.jhu.edu`