Math 401: Introduction to Abstract Algebra
Problem Set 5
due: March 5, 2019

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Read. §2, §3, §4

Exercise 1. Prove that $(gh)^{-1} = h^{-1}g^{-1}$ for all elements g, h in a group G.

Exercise 2. For a fixed value of $n \in \mathbb{N}_{>0}$ verify that the set $\{0, 1, \ldots, n-1\}$ defines a group under the operation "addition modulo n."

Exercise 3. A **partition** on a set A is a decomposition of A into disjoint subsets whose union is A.

(i) An equivalence relation $\sim$ on a set A defines a partition of A into disjoint subsets, namely the equivalence classes of the equivalence relation. Describe this construction in your own words.
(ii) Conversely, given any partition of A into disjoint subsets whose union is A, define a corresponding equivalence relation $\sim$ whose equivalence classes coincide with these subsets.

Thus, the set of partitions of A is isomorphic to the set of equivalence relations on A.

Exercises.
§3 | 3.3, 3.4

Exercise 4. The multiplication operation $- \cdot -\colon G \times G \to G$ for a group G can be specified by writing down its **multiplication table**: the columns and the rows are each labelled by the elements of G, and then the entry in row g and column h is the product $g \cdot h$.

G	e	g	h	$\cdots$	k
e	e	g	h	$\cdots$	k
g	g	g^2	$g \cdot h$	$\cdots$	$g \cdot k$
h	h	$h \cdot g$	h^2	$\cdots$	$h \cdot k$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
k	k	$k \cdot g$	$k \cdot h$	$\cdots$	k^2

(i) Explain the pattern that you see in the first row and first column of the table (indexed by the identity element e).
(ii) Prove that every row and every column of the multiplication table of a group contains all elements of that group exactly once (like a Sudoku diagram).

Exercise 5.

(i) Sketch a proof that the unit circle centered at the origin in $\mathbb{R} \times \mathbb{R}$ defines a group with identity element $(1, 0)$ and with addition defined by "adding angles," where the angle of a unit vector is measured counterclockwise starting from the positive x axis.
(ii) Is this group abelian?
(iii) Can this group be described as the symmetry group of a geometric figure?

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