Math 401: Introduction to Abstract Algebra

Problem Set 4[1]

due: February 25, 2019

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Read. §1, §2, §3

Exercise 1.

(i) Express all eight symmetries of the square as composites of the 90° clockwise rotation and the reflection through the north/south axis that bisects two of the sides.

(ii) Prove that the 90° clockwise rotation σ and the reflection through the north/south axis ρ do not *commute*: that is, show that the symmetry $\rho \cdot \sigma$ you get by first performing the rotation and then the reflection is not equal to the symmetry $\sigma \cdot \rho$ you get by first performing the reflection and then the rotation.[2]

Exercise 2.

(i) Explain how a permutation of the faces of a tetrahedron determines a permutation of the vertices of the tetrahedron.

(ii) Explain how a permutation of the faces of a tetrahedron determines a permutation of the edges of the tetrahedron.

Exercise 3. Develop a scheme for describing all of the symmetries of the cube and determine how many symmetries there are.[3]

Exercises. [4]

§1 | 1.8, 1.9

Exercise 4. Prove that $(gh)^{-1} = h^{-1}g^{-1}$ for all elements g, h in a group G.

Exercise 5. For a fixed value of $n \in \mathbb{N}_{>0}$ verify that the set $\{0, 1, \ldots, n-1\}$ defines a group under the operation "addition modulo n."

Exercise 6*. Verify that there are 60 symmetries of the regular dodecahedron. Since $60 \mid 120 = 5!$ this suggests that each symmetry of the dodecahedron might be understand as a permutation of some five element set. Find such a set.

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[1] Problems labelled n* are optional (fun!) challenge exercises that will not be graded.

[2] Since we think of symmetries as similar to funtions, we read the composite of two symmetries from right to left, in "composition order."

[3] A symmetry of the cube is not allowed to "turn the cube inside out."

[4] Exercise 1.8 refers to 24 symmetries of the hexagonal plate, but some of these involve "turning a 3D figure inside-out," a rigid motion we prohibited in 3D when discussing the symmetries of the tetrahedron in class." I'll let you choose how you want to approach this problem. Either continue to prohibit "turning a 3D figure inside out" (in which case part of the problem is to determine how many symmetries the hexagonal plate has) or solve the problem as written, including the inversions as symmetries.