Math 401: Introduction to Abstract Algebra
Practice Midterm
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Conventions: In what follows $\mathbb{Z}$ denotes the group of integers under addition, $\mathbb{Z}/n$ denotes the cyclic group under addition modulo n, D_n denotes the dihedral group of symmetries of a regular n-gon, and S_n denotes the symmetric group of permutations of n elements.

True or False

(1 point) Indicate whether each of the following statements is true or false (circle one).
(2 points) For each true statement, give a short (one to two sentence) justification, explaining the essential reason for its correctness; for each false statement, provide either a counter-example or, if a counter-example would not make sense, a short disproof.

1. (T or F) The set of all 2×2 matrices with real coefficients forms a group under matrix multiplication.

2. (T or F) The Klein four group is abelian.

3. (T or F) The dihedral group D_5 is generated by the 72° rotation through the center of mass of the pentagon.

4. (T or F) Every non-zero[1] subgroup of $\mathbb{Z}$ is cyclic.

[1]Technically, the trivial group with a single element *is* a cyclic group, but we're excluding it to avoid confusion.

5. (T or F) Let $\phi: G \to H$ define a group homomorphism and consider two elements $g_1, g_2 \in G$. If $g_1 g_2 = g_2 g_1$ in G, then $\phi(g_1)\phi(g_2) = \phi(g_2)\phi(g_1)$ in H.

6. (T or F) Let $\phi: G \to H$ define a group homomorphism and consider two elements $g_1, g_2 \in G$. If $\phi(g_1)\phi(g_2) = \phi(g_2)\phi(g_1)$ in H, then $g_1 g_2 = g_2 g_1$ in G.

7. (T or F) There exists a non-zero homomorphism $\mathbb{Z}/17 \to \mathbb{Z}$.

8. (T or F) For $2 \leq k \leq n$ a k-cycle $(a_1 \cdots a_k) \in S_n$ is an even permutation if and only if k is odd.