MATH 301: INTRODUCTION TO PROOFS

EMILY RIEHL

Instructor:

- Emily Riehl, eriehl@math.jhu.edu, she/her

TA:.

- Anthony Agwu, aagwu1@jhu.edu, he/him

Lectures: MW 1:30-2:45, Mergenthaler 111 / Zoom

Section: F 1:30-2:20 on Zoom

Textbook:

- The primary class textbook will be *An Infinite Descent into Pure Mathematics*, a book under development by Clive Newstead, which is freely available at infinitedescent.xyz/ (choose version 0.4 the "stable download")
- An optional secondary resource is *How To Prove It: A Structured Approach*, Daniel J. Velleman, which was used as the course text in the previous iteration of this class.

Course website:

- Grades etc will be posted on the Blackboard website for AS.110.301.01.SP21 Introduction to Proofs
- Problem sets and supplemental materials can be found at www.math.jhu.edu/∼eriehl/301

What you can call me. You are welcome to address me as "Professor Riehl," "Dr. Riehl," or "Emily." I use she/her pronouns.

What will be different this semester? My goal will be to teach you the same amount of math as I would in a normal semester, so in particular, I'll cover roughly the same topics in lecture as I have in the past. But I think it's unreasonable to expect the same productivity in a pandemic, so my plan is to assign fewer problems on each problem set. Each week, I'll mark a problem or two as "just for fun." These won't be graded (as I'm also trying to decrease the workload of our TA).

Classroom Climate. I am committed to creating a classroom environment that values the diversity of experiences and perspectives that all students bring. Everyone here has the right to be treated with dignity and respect. I believe fostering an inclusive climate is important because research and my experience show that students who interact with peers who are different from themselves learn new things and experience tangible educational outcomes. Please join me in creating a welcoming and vibrant classroom climate. Note that you should expect to be challenged intellectually by me, David, and your peers, and at times this may feel uncomfortable. Indeed, it can be helpful to be pushed sometimes in order to learn and grow. But at no time in this learning process should someone be singled out or treated unequally on the basis of any seen or unseen part of their identity.

If you ever have concerns in this course about harassment, discrimination, or any unequal treatment, or if you seek accommodations or resources, I invite you to share directly with me or David. I promise that we will take your communication seriously and to seek mutually acceptable resolutions and accommodations. Reporting will never impact your course grade. You may also share concerns with the department chair (David Savitt, savitt@math.jhu.edu), the Director of Undergraduate Studies (Richard Brown, brown@math.jhu.edu), the Assistant Dean for Diversity and Inclusion (Darlene Saporu, dsaporu@jhu.edu), or the Office of Institutional Equity (oie@jhu.edu). In handling reports, people will protect your privacy as much as possible, but faculty and staff are required to officially report information for some cases (e.g. sexual harassment).

Date: Spring 2021.

Personal Wellbeing.

- If you are sick, in particular with an illness that may be contagious, notify me by email and you will be excused from coming to class. Rather, visit the Health and Wellness Center: 1 East 31 Street, 410-516-8270. See also studentaffairs.jhu.edu/student-life/support-and-assistance/absences-from-class/illness-note-policy
- All students with disabilities who require accommodations for this course should contact me at their earliest convenience to discuss their specific needs. If you have a documented disability, you must be registered with the JHU Office for Student Disability Services (385 Garland Hall; 410-516-4720; web.jhu.edu/disabilities) to receive accommodations.
- If you are struggling with anxiety, stress, depression or other mental health related concerns, please consider visiting the JHU Counseling Center. If you are concerned about a friend, please encourage that person to seek out our services. The Counseling Center is located at 3003 North Charles Street in Suite S-200 and can be reached at 410-516-8278 and online at studentaffairs.jhu.edu/counselingcenter/.

Support.

> At key times, it is more useful to take stock of what one knows than blindly march forward hoping for the best. A difficulty at this time signals the need to reread the previous material carefully. *If the mystery persists, that's what office hours are there for.* But typically you should be able to find your way out on your own, based on the information we have given you, and you will most likely learn more this way. You should give it your best try before seeking professional help. -Paolo Aluffi, *Algebra: Chapter 0*, §I.3

If you are stuck on a problem on the homework or confused about something that happened in class, my first recommendation is to ask one of your classmates. If they know the answer, you'll give them an invaluable opportunity to reinforce their knowledge by putting it into words. If they don't, chances are you'll be able to figure it out together, and both learn more via the process of self-discovery.

If you need further guidance, the Math Help room is open from approximately 9am-9pm Monday-Thursday and 9am-5pm on Friday on Zoom, and I assure you that the graduate students who staff it would much rather talk about mathematical proofs than calculus. Anthony and I will also hold office hours at a time to be determined.

The structure of the course. This course will have two phases.

Phase I: constructing mathematical proofs

For the first eight weeks, the format will be that of a traditional lecture course. The following schedule of lectures is aspirational and subject to change.

- January 25: propositional logic (§1.1) + skim "getting started" if your math is a bit rusty
- January 27: proof-writing (§1.1, appendix A)
- February 1: variables and quantifiers (§1.2)
- February 3: logical equivalence (§1.3)
- February 8: tautologies and more proof strategies (§1.3)
- February 10: sets and set operations (§2.1, §2.2)
- February 15: functions (§3.1)
- February 17: injections and surjections (§3.2)
- February 22: Peano's axioms (§4.1)
- February 24: weak induction (§4.2)
- March 1: strong induction (§4.3)
- March 3: finite sets (§7.1)
- March 8: constructive mathematics and computer proof assistants I
- March 10: constructive mathematics and computer proof assistants II
- March 15: countable and uncountable sets (§9.1)
- March 17: cardinality and cardinal arithmetic (§9.2-9.3)

Problem Sets. During the first part of the course, a problem set will be due each Wednesday in class, starting on Wednesday, February 3, and cover the material discussed during the previous week. There will be seven problem sets in total. At the end of the semester, the lowest problem set grade will be dropped. If there is a week during which you are simply too

busy to turn in the homework on time, do not panic. It's not a big deal to take a zero one week, especially if it's just one week, because that problem set will automatically be dropped.

Collaboration on written homework is allowed and encouraged. However, each student must write up their solutions to the problems individually and in their own words, and must acknowledge their collaborators by name on their written assignments. Copying from another student or any other source is prohibited. The policies of the Johns Hopkins Ethics Guide will apply to this course: http://e-catalog.jhu.edu/undergrad-students/student-life-policies/

Infinitesimal extra credit. The course textbook *An Infinite Descent into Pure Mathematics* is a work in progress and the author would like feedback from readers about their experience using the text. In the class you can earn infinitesimal extra credit by writing Clive Newstead an email containing constructive suggestions or comments based on your experience reading the text. Infinitesimal extra credit comes into play if your final numerical grade falls just under the cutoff for the next letter grade. In that circumstance, adding a small amount, such as you would earn through infinitesimal extra credit, might boost you over the line. For the credit to count, please either cc or forward this to me by April 30th. Note that for the majority of the class, probabilistically speaking, the presence or absence of infinitesimal extra credit will make no difference whatsoever, so this is not a thing to worry about.

Phase II: practicing mathematical proofs

In the four weeks after Spring "Break,"[1] we will study *metric spaces*, which are sets of points equipped with a "distance function" satisfying some natural axioms. Importantly, the classroom will be "flipped": in place of the traditional lecture format we will adopt an "inquiry-based learning" (IBL) approach.

In an IBL classroom, the students, never the instructor, stand at the metaphorical chalkboard. In lieu of lectures, the class develops its own text over the course of the term, following a "script" supplied by the instructor (a carefully chosen sequence of definitions, examples, lemmas, and theorems) and filling in their own proofs, completed as homework, and then presented and critiqued during the class meetings. The aim is to give you an ample opportunity to write your own proofs and also practice evaluating whether an argument supplied by yourself or presented by a classmate is complete and correct. I do not expect student presentations to be perfect — to the contrary, I hope they are not! It's much harder to learn how to write a precise proof if the only examples you ever see are polished. Potential logical pitfalls are much more effectively illuminated if someone walks right into them, so I hope that there will be many mistakes made that we can collectively learn from as a class.

Class discussions and class presentations are major components of this phase of the course. Students are expected to be active participants in the classroom, and are expected to conduct themselves with professionalism and respect for their classmates. Our goal is to create a supportive classroom environment where students are comfortable testing ideas, and both offering and receiving constructive criticism from peers.

Metric Spaces Workbook. There will be no textbook during the inquiry based learning part of the course, the goal being to actively develop the theory of metric spaces as a class. Each student will fill in the proofs in a script, to be supplied by the instructors, producing a *workbook*, which will combine the textbook, lecture notes, and homework for this part of the course. Please do not consult any supplemental material (eg Wikipedia) outside of what is handed-out in class. Workbooks will be collected for grading approximately once a week (April 5, April 12, April 26th).

Blackboard presentation requirement. During the inquiry based learning stage of the course, every student will be asked to present their arguments to the class, either in Mergenthaler 111 or on Zoom. Presenters will be randomly selected. It's okay to pass when your name is called, but the expectation is that each student will present at the board once a week, with at least four presentations in total. Any student who presents four proofs at the board will receive full credit for the presentation requirement, whether or not these proofs are completely correct.

Metric spaces exam. An oral exam, covering the metric space unit, will be held on Monday, April 26th. You will be asked to explain some of the key concepts from the metric space unit and illustrate them with examples. The aim is to encourage you to work to *understand*, rather than simply transcribe, the ideas being discussed. If this makes you nervous, try to find someone who has taken a class with me in the past, who will reassure you that this exam format is very forgiving and tends to go well for most students.

[1] I get that this is a joke this year. We will not have class on March 22th or April 14th. The metric space unit will start on March 24th.

Closing discussion: deconstructing mathematical proofs

During the final class meeting, we will shift to a seminar format and discuss the way proof functions in the mathematical community and in society at large. We'll consider case studies that question whether mathematical arguments can be too long or too complicated to count as "proof" and discuss the role played by axioms. These debates will be informed by excerpted essays assigned as reading.

During the seminar phase of the course, there will be no more problem sets and no new proofs. Instead, there will be a class participation requirement, namely that each student make one comment. If for some reason you have to miss class, the class participation requirement can be satisfied by emailing me a two paragraph reading response within 24 hours of the class meeting.

Grades. A numerical grade will be assigned based on the following formula:

- 1/2 problem sets

plus

- 1/4 metric spaces workbook (1/5th of this for completeness and 4/5th awarded for the writing itself)
- 1/10 blackboard presentation (full credit if blackboard presentation requirement is satisfied)
- 1/10 midterm

totaling 95% of your final grade, plus

- 1/20 class participation (full credit received if the class participation requirement is satisfied)