Math 301: Introduction to Proofs
Problem Set 5
due: March 4, 2019

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Read. §3.6, §3.7, §6.1, §6.2, §6.4

Exercises.

§3.6	12
§3.7	6

Exercise 1. For positive integers a and b, we say a **divides** b and write $a \mid b$ if there exists an integer k so that $b = ka$. The following exercises are "true or false". If true, supply a proof. If false, supply a counterexample.

(a) $\forall n \in \mathbb{N}$, $n \mid 0$.
(b) $\forall n \in \mathbb{N}$, if $0 \mid n$ then $n = 0$.
(c) If $a \mid b$ and $b \mid c$ then $a \mid c$.
(d) If $a \mid c$ and $b \mid c$ then $(a \cdot b) \mid c$.

Exercise 2. For $a, b \in \mathbb{N}$, the **least common multiple** $\text{lcm}(a, b)$ is the smallest integer so that $a \mid \text{lcm}(a, b)$ and $b \mid \text{lcm}(a, b)$. The **greatest common divisor** $\gcd(a, b)$ is the largest integer so that $\gcd(a, b) \mid a$ and $\gcd(a, b) \mid b$. The following exercises are "true or false". If true, supply a proof. If false, supply a counterexample.[1]

(a) $\forall a, b \in \mathbb{N}$, if $\gcd(a, b) = 0$ then $a = b = 0$.
(b) $\forall a, b \in \mathbb{N}$, $\gcd(a, b) \mid \text{lcm}(a, b)$
(c) $\forall a, b \in \mathbb{N}$, if $d \mid a$ then $d \mid \gcd(a, b)$
(d) $\forall a, b \in \mathbb{N}$, if $d \mid a$ then $d \mid \text{lcm}(a, b)$
(e) $\forall a, b \in \mathbb{N}$, if $d \mid a$ and $d \mid b$ then $d \mid \gcd(a, b)$
(f) $\forall a, b \in \mathbb{N}$, if $d \mid a$ and $d \mid b$ then $d \mid \text{lcm}(a, b)$
(g) For all $a, b \in \mathbb{N}$, $ab = \text{lcm}(a, b) \cdot \gcd(a, b)$.

As a check, ensure that your answers for these are consistent — if the truth of one implies the truth of another, be sure that this is possible!

Exercise 3. Find a formula for the sum $1 + 3 + 5 + \cdots + (2n - 1)$ of the first $n + 1$ odd numbers (for n a positive integer) and prove (by induction) that your formula is correct.[2]

Exercises.

§6.1	2
§6.2	10
§6.4	16, 17

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[1] Hint: some of these are made easier by applying results of exercise 1 above.

[2] Hint: To discover the formula, first compute this sum for some particular small values of n and look for a pattern.