Math 301: Introduction to Proofs
Problem Set 4
due: February 25, 2019

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Read. §3.3, §3.4, §3.5

Exercise 1. Let $f\colon \mathbb{R} \to \mathbb{R}$ be a function. Negate the following expression

$$\forall x \in \mathbb{R}, \forall \epsilon > 0, \exists \delta > 0, \forall y \in \mathbb{R}, |x - y| < \delta \to |f(x) - f(y)| < \epsilon.$$

Exercise 2 (Drinker's paradox[1])**.** We will explore yet another manifestation of the quirks of classical logic. In this exercise we will prove the seemingly bizarre statement that, for an arbitrary pub, "there is someone in the pub such that if that person is drinking then so is everyone else."

(a) Let X be the collection of people in the pub, and let $P(x)$ be the statement "x is drinking". Transform our claim into a logical statement using quantifiers and implication.[2]

(b) By making use of the Law of the Excluded Middle on the predicate $P(x)$

($\star$) either "$\forall x \in X, P(x)$" is true or "$\forall x \in X, P(x)$" is false

consider whether everyone in the pub is drinking and derive a proof from there.

Exercise 3. Prove that for all non-zero rational numbers q, there exists a non-zero integer m, so that mq is a positive integer.

Exercises.
§3.3 | 1, 14, 18, 19

Exercise 4. Prove that for all integers n, $n(n+1)$ is even.

Exercises.
§3.5 | 10, 20, 21, 30

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[1] This is not truly a paradox, but it sounds improbable when phrased in natural language, likely because we automatically infer a sense of causality.

[2] Be careful to avoid overloading your chosen names for variables.