Math 301: Introduction to Proofs
Problem Set 3
due: February 18, 2019

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Read. §2.3, §4.1, §3.1, §3.2

Definition. A function $f: A \to B$ from a set A to a set B is an assignment or "mapping" from elements $a \in A$ to elements $b \in B$ such that

1. Every element $a \in A$ is assigned an element of B
2. Each element $a \in A$ is assigned to only one element of B

When these conditions are satisfied we write $f(a)$ for the element of B assigned to the element $a \in A$ by f. We may also write $a \mapsto b$ to indicated that the element $a \in A$ is assigned the element $b \in B$.

Exercise 1. Write out the two conditions of an assignment f being a function using quantifiers.

Exercise 2. Decide whether the following are functions, giving reasons for your answers:

(a) The assignment $f: \mathbb{Z} \to \mathbb{Z}$ given by $f(x) = -2x$
(b) The assignment $g: \mathbb{N} \to \mathbb{N}$ given by $n \mapsto$"the (n+1)$^{\text{th}}$ prime number"
(c) The assignment $h: \mathbb{Q} \to \mathbb{Q}$ given by $h(y) = 1/y$
(d) The assignment $k: \mathbb{N} \to \mathbb{N}$ given by $m \mapsto d$ where d is a divisor of m.

Exercise 3. Suppose A and B are finite sets, where the number of elements in A is α and the number of elements in B is β.

(a) How many elements are in the set $A \times B$?
(b) How many elements are in the set $A \sqcup B$?
(c) How many elements are in the set B^A of functions from A to B?

In light of the answers to (i), (ii), and (iii) explain why some authors prefer to use "$A + B$" as notation for the disjoint union of two sets.

Exercise 4. Write $2 = \{\bot, \top\}$ or $2 = \{0, 1\}$ for the set with two elements. (Your choice which notation you want to use for its elements.)

(a) Let $S \subset A$. Define a function $\chi_S \colon A \to 2$ that is related to S in some natural way.
(b) Use part (a) to define a natural function $\chi \colon P(A) \to 2^A$.
(c) Show that the function $\chi \colon P(A) \to 2^A$ that you've defined in part (b) is a bijection.[1]

Exercise 5. Write $2 = \{\ell, r\}$ for the set with two elements. Define a bijective function $A \times A \to A^2$ and prove that it is a bijection.[2]

Exercises.

§2.3	9, 10, 12

Exercises.

§3.2	1, 2

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[1]If the function you've defined in part (b) is not a bijection, you might need to redefine the function χ.

[2]Here $A \times A$ denotes the cartesian product of A with itself, while A^2 is the set of functions $2 \to A$. On account of this bijection it's reasonable to use the notation A^2 for $A \times A$.