MATH 161: CATEGORY THEORY IN CONTEXT

EMILY RIEHL

COURSE DESCRIPTION: An introduction to categories, functors, natural transformations, the Yoneda lemma, limits and colimits, adjunctions, monads, and other topics as time permits with the aim of revisiting a broad range of mathematical examples from the categorical perspective.

OVERVIEW

Atiyah described mathematics as the "science of analogy"; in this vein, the purview of category theory is *mathematical analogy*. Specifically, category theory provides a mathematical language that can be deployed to describe phenomena in any mathematical context. Perhaps surprisingly given this level of generality, these concepts are neither meaningless and nor in many cases so clearly visible prior to their advent. In part, this is accomplished by a subtle shift in perspective. Rather than characterize mathematical objects directly, the categorical approach emphasizes the morphisms, which give comparisons between objects of the same type. Structures associated to particular objects can frequently be characterized by their *universal properties*, i.e., by the existence of certain canonical morphisms to or from other objects of a similar form.

A great variety of constructions can be described in this way: products, kernels, and quotients for instance are all *limits* or *colimits* of a particular shape, a characterization that emphasizes the universal property associated to each construction. Tensor products, free objects, and localizations are also uniquely characterized by universal properties in appropriate categories. Important technical differences between particular sorts of mathematical objects can be described by the distinctive properties of their categories: that they have certain limits and colimits, but not others, that certain classes maps are *monomorphisms* or *epimorphisms*. Constructions that take one type of mathematical object to objects of another type are often morphisms between categories, called *functors*. In contrast with earlier numerical invariants in topology, functorial invariants (such as the fundamental group) tend both to be more easily computable and also provide more precise information. Functors can then be said to *preserve* particular categorical structures, or not. Of particular interest is when a functor describes an *equivalence* of categories, which means that objects of the one sort can be translated into and reconstructed from objects of another sort.

Date: Spring 2015.

Category theory also contributes new proof techniques, such as *diagram chasing* or duality; Steenrod called these methods "abstract nonsense."[1] The aim of this course will be to introduce the language, philosophy, and basic theorems of category theory. A complementary objective will be to put this theory into practice: studying functoriality in algebraic topology, naturality in group theory, and universal properties in algebra.

So why study category theory?

- It's fun, and elegant.
- It provides a useful organizing principle, which can make new mathematical ideas easier to learn and familiar concepts easier to remember:

> The aim of theory really is, to a great extent, that of systematically organizing past experience in such a way that the next generation, our students and their students and so on, will be able to absorb the essential aspects in as painless a way as possible, and this is the only way in which you can go on cumulatively building up any kind of scientific activity without eventually coming to a dead end. — Atiyah "How research is carried out"

- It will give us a chance to explore some really deep ideas, e.g., of *representability*, that are nonetheless relatively accessible. My hope is that you will leave this course feeling more in command of the mathematics that you know.

General information

Lectures.
TTh 11:30am-1pm, Science Center 304

Course website.
http://www.math.harvard.edu/~eriehl/161

Contact.
Email: eriehl@math.harvard.edu
Office: Science Center 320

Office hours.
Monday 3-4, immediately following each lecture, or by appointment

Textbook.
The main text will be comprised of lecture notes, to be posted on the course website. For supplementary reading, I recommend *Basic Category Theory* by Tom Leinster, which will be available at the Coop.

Course Assistant.
Michael Fountaine, mfount@college.harvard.edu

Prerequisites

Math 123 (may be taken concurrently) and Math 131, or permission of the instructor. The prerequisites are designed to guarantee a level of mathematical sophistication sufficient to appreciate the examples that will be discussed. In the absence of this, extraordinary enthusiasm and motivation will likely suffice.

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[1] Contrary to popular belief, this was not intended as an epithet.

Assessments

Problem sets. Problem sets will be posted on Thursday afternoons and due in class on the following Tuesday. At the end of the semester, the lowest grade on a problem set will be dropped. In light of this policy, late work will very seldom be accepted.

Collaboration. Collaboration on problem sets is encouraged but each student must write up his or her solutions individually. To get the most out of this course, it is recommended that you first think about the problems on your own before working with a group. Please acknowledge any collaborators by writing their names on your problem sets.

Oral assignments. In addition to the weekly problem sets, there will be a handful of oral assignments, concentrated within the first third of the course. The greatest challenge in aclimating to the categorical viewpoint comes right at the beginning, with the introduction of natural transformations, representable functors, and the Yoneda lemma. The oral assignments will be designed so that the expected confusion does not persist for very long. Oral presentations will take place at the chalkboard and will ordinarily require no more than 10 minutes. The grading scheme will be pass (full credit) or fail (no credit). Passing signifies that the student has succeeded in communicating a reasonable understanding of the assigned material. Failed oral examinations may be retaken as many times as necessary within one week of the original assignment.

Extra credit. Extra credit, worth 1% of the final grade, will be rewarded to any student who brings me an example of a mathematical application of a categorical idea that (a) is not already described in the course lecture notes and (b) has not yet been told to me by anyone else. This exercise may be repeated as often as desired, but only 3% extra credit may be earned at maximum.

Exams. There will be one midterm examination, to be given in class on Tuesday, March 31st. The exam will draw heavily from material discussed in class. The purpose of the exam is to provide an opportunity to pause and reflect, with the hopes of facilitating long-term understanding. Also in support of this goal, lecture attendance is highly encouraged. The final, given during the standard time slot, will have a similar length and format.

Course grades. A numerical grade will be assigned based on the following formula: 40% problem sets, 10% oral assignments, 25% midterm, 25% final. At minimum, letter grades will be the standard conversion (e.g. A^- for 90%) from the numerical score computed by this formula.

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