**Math 161: Category theory in context**
Problem Set 6
due: April 7, 2015

Emily Riehl

**Exercise 1.** Prove Lemma 4.1.2 and Remark 4.1.4.

**Exercise 2.** Define left and right adjoints to $\mathrm{ob}\colon \textsc{Cat} \to \textsc{Set}$ and to $\mathrm{Vert}\colon \textsc{Graph} \to \textsc{Set}$, the functor that takes a graph to its vertex set, for either directed or undirected graphs, as you prefer.

**Exercise 3.** Explain each step needed to convert the statement of Lemma 4.2.1 into the statement of Lemma 4.2.2.

**Exercise 4.** Suppose given a pair of functors $F\colon \mathcal{C} \rightleftarrows \mathcal{D}\colon G$ together with unit and counit natural transformations satisfying the triangle identities. Prove the converse direction of Proposition 4.2.4 by using the unit and counit to define a natural bijection $\mathcal{D}(Fc, d) \cong \mathcal{C}(c, Gd)$.

**Exercise 5.** When does the functor $!\colon \mathcal{C} \to \mathbb{1}$ have a left adjoint? When does it have a right adjoint?

**Exercise 6.** Pick your favorite forgetful functor from Example 4.1.9 and prove that it is a right adjoint by defining its left adjoint, the unit, and the counit, and demonstrating that the triangle identities hold.

**Exercise 7.** Each component of the counit of an adjunction is a terminal object in some category. What category?

Dept. of Mathematics, Harvard University, 1 Oxford Street, Cambridge, MA 02138
*E-mail address*: eriehl@math.harvard.edu