MIDTERM FOR 110 405
FALL 2003

Answer all six questions. The first two questions are True/False. Fully justify your answer for the last four questions.

Question 1. (10 points; True/False). The sequence

$$x_n = \sum_{i=1}^{n} \frac{1}{i}$$

is Cauchy.

Question 2. (10 points; True/False). The sequence

$$y_n = \sum_{i=1}^{n} 2^{-i}$$

is Cauchy.

Question 3. (20 points). Let T be the set of all Taylor series with 0's or 1's as coefficients, i.e., an element of T is an infinite series like

$$a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n + \cdots$$

where each a_i is either 0 or 1. Is T countable?

Question 4. (20 points). Suppose that x_n is a Cauchy sequence. Prove that there exists a number $N \in \mathbf{N}$ so that $|x_j| \leq N$ for every j.

Question 5. (20 points). Let A be a set. Show that if x is a limit point of A, then there exists a sequence x_n of distinct points in A which converge to x.

Question 6. (20 points). Suppose that $S \subset \mathbf{R}$ is an uncountable set. Prove that S contains at least one of its limit points.

Prof. Minicozzi.