MIDTERM FOR 110 405
FALL 2004

Answer all five questions. The first two questions are short answer. Fully justify your answer for the last three questions.

Question 1. (10 points; True/False). Every bounded sequence has a convergent subsequence; no proof is required.

Question 2. (10 points; Short answer). Give an example of a set that is both open and closed; no proof is required.

Question 3. (25 points). Suppose that E is a non-empty compact set. Show that $\sup(E)$ is contained in E.

Question 4. (25 points). Suppose that for each λ in a set Λ, we have a positive real number $a_\lambda > 0$. Suppose also that for any natural number n and any $\lambda_1, \ldots, \lambda_n \in \Lambda$ we have

$$\sum_{i=1}^{n} a_{\lambda_i} < 1 \, .$$

Prove that the set Λ is at most countable (i.e, is either countable or finite).

Question 5. (30 points). Suppose that a sequence y_n is defined iteratively by $y_0 = 1$ and then

$$y_{n+1} = \frac{1}{2 + y_n} \, .$$

(a) Compute the next three terms y_1, y_2, and y_3.
(b) Prove that the sequence y_n converges.

Prof. Minicozzi.