HOMEWORK PROBLEM SET 8: DUE APRIL 3, 2019

AS.110.211 HONORS MULTIVARIABLE CALCULUS
PROFESSOR RICHARD BROWN

Question 1. Answer the following in an expository fashion, explaining fully.

(a) What is the nature of the flaw in **Definition 14.1** of **Lecture 14**, defining a definite double integral of a function f on a region $\mathcal{R}$ in terms of a limit, taken as the partition size in the two variables grows toward infinity. The problem is that, given the definition as stated, even for continuous positive functions, this limit might not converge to the volume of the solid formed between $\mathcal{R}$ and $\mathbf{graph}(f)$. Why not?

(b) Why is it true that one way to find the volume of a solid $\mathcal{S} \in \mathbb{R}^n$ is to simply integrate the constant function $f(\mathbf{x}) = 1$ over $\mathcal{S}$?

Question 2. In **Example 14.6** of **Lecture 14**, we broke up the nonelementary annular region $\mathcal{D}$ into four elementary regions, all of Type I. Rewrite $\mathcal{D}_1$ and $\mathcal{D}_4$ as elementary regions of Type II.

Question 3. Without explicitly evaluating integrals, determine by argument the value of the following integrals:

(a) $\displaystyle\iint_{\mathcal{D}} x^3\, dA$, for $\mathcal{D} \subset \mathbb{R}^2$ the region defined by the inequality $0 \leq y \leq 4 - x^2$.

(b) $\displaystyle\iint_{\mathcal{R}} (x^5 + 2y)\, dA$, for $\mathcal{R} = [-3, 3] \times [-2, 2]$.

Question 4. Do the following:

(a) Evaluate $\displaystyle\int_0^2 \int_{\frac{x^2}{4}}^{\frac{x^2}{2}} (x^2 + y^2)\, dy\, dx$.

(b) Evaluate $\displaystyle\iint_{\mathcal{D}} xy\, dA$, for $\mathcal{D}$ the region bounded by $x = y^3$ and $y = x^2$.

(c) Find the volume of the solid lying under the plane $z = 24 - 2x - 6y$ and over the region in the xy-plane bounded by $y = 4 - x^2$, $y = 4x - x^2$ and the y-axis.

(d) Show, for $\mathcal{R} = [a, b] \times [c, d]$, where f is continuous on $[a, b]$ and g is continuous on $[c, d]$, that

$$\iint_{\mathcal{R}} f(x)g(y)\, dA = \left(\int_a^b f(x)\, dx \right) \left(\int_c^d g(y)\, dy \right).$$

Question 5. Reverse the order of integration of the following and evaluate:

(a) $\displaystyle\int_0^2\int_0^{4-y^2} x\,dx\,dy.$

(b) $\displaystyle\int_{-2}^1\int_{x^2-2}^{-x} (x-y)\,dy\,dx.$

(c) $\displaystyle\int_0^1\int_{3y}^3 \cos(x^2)\,dx\,dy.$ (If you are ambitious, evaluate this integral without switching the order of integration.)

Question 6. Evaluate the following:

(a) $\displaystyle\int_1^3\int_0^z\int_1^{xz} (x+2y+z)\,dy\,dx\,dz.$

(b) $\displaystyle\iiint_{\mathcal{W}} z\,dV$, for $\mathcal{W}$ the region in the first octant bounded by the cylinder $y^2+z^2=9$ and the planes $y=x$, $x=0$, and $z=0$.