HOMEWORK PROBLEM SET 11: DUE APRIL 24, 2019

AS.110.211 HONORS MULTIVARIABLE CALCULUS
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Question 1. Do the following:

- **(a)** Find the flux of $\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ across the surface $\mathcal{S}$ consisting of the triangular region of the plane $2x - 2y + z = 2$ that is cut out by the coordinate planes. Use the upward-pointing normal to orient $\mathcal{S}$.
- **(b)** Find $\iint_{\mathcal{C}} (x^2 + y^2)\, dS$, where the surface $\mathcal{C}$ is a cylinder of height $h > 0$ and radius $a > 0$ centered on the z-axis. $\mathcal{C}$ has neither a top nor a bottom.

Question 2. For $\mathcal{S} \in \mathbb{R}^3$ the closed surface called a canister; a cylinder $x^2 + y^2 = 9$, along with a flat top at $z = 4$ and a flat bottom at $z = 0$, oriented outward, determine the following:

- **(a)** $\displaystyle\oiint_{\mathcal{S}} xyz\, dS$.
- **(b)** $\displaystyle\oiint_{\mathcal{S}} (x\,\mathbf{i} + y\,\mathbf{j}) \bullet d\mathbf{S}$.

Question 3. Find the flux of $\mathbf{F} = x^2\,\mathbf{i} + xy\,\mathbf{j} + xz\,\mathbf{k}$ across the upper hemisphere $x^2 + y^2 + z^2 = a^2$, for $z \geq 0$. Orient the hemisphere with an upward pointing normal.

Question 4. Let $\mathcal{S} \in \mathbb{R}^3$ be the funnel-shaped surface defined by $x^2 + y^2 = z^2$ for $1 \leq z \leq 9$, and $x^2 + y^2 = 1$ for $0 \leq z \leq 1$.

- **(a)** Sketch $\mathcal{S}$.
- **(b)** Determine outward pointing normal vectors to $\mathcal{S}$.
- **(c)** Evaluate $\displaystyle\iint_{\mathcal{S}} \mathbf{F} \bullet d\mathbf{S}$, for $\mathbf{F} = -y\,\mathbf{i} + x\,\mathbf{j} + z\,\mathbf{k}$ with the outward-pointing orientation.

Question 5. Do the following:

- **(a)** Verify Stokes' Theorem for $\mathcal{S}$ given by $x^2 + y^2 + 5z = 1$, $z \geq 0$, oriented upward, and $\mathbf{F} = xz\,\mathbf{i} + yz\,\mathbf{j} + (x^2 + y^2)\,\mathbf{k}$.
- **(b)** Verify Gauss' Theorem for $\mathbf{F} = x^2\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$, and

$$\mathcal{W} = \left\{ (x, y, z) \in \mathbb{R}^3 \;\middle|\; x^2 + y^2 + 1 \leq z \leq 5 \right\}.$$

Question 6. Verify that Stokes' Theorem implies Green's Theorem. (Hint: When assuming Stokes' holds, assume that $\mathbf{F} = M(x,y)\,\mathbf{i} + N(x,y)\,\mathbf{j}$ is both independent of z and has no $\mathbf{k}$-component.

Question 7. Let $\mathcal{S}$ be the surface defined by $z = 4 - 4x^2 - y^2$, $z \geq 0$, oriented with nonnegative $\mathbf{k}$-component. For $\mathbf{F} = x^3\,\mathbf{i} + e^{y^2}\,\mathbf{j} + ze^{xy}\,\mathbf{k}$, find $\iint_{\mathcal{S}} \nabla \times \mathbf{F} \bullet d\mathbf{S}$. (Hint: Argue that you can integrate over a different surface.)

Question 8. Use Gauss' Theorem to find the volume of the solid bounded by the paraboloids $z = 9 - x^2 - y^2$, and $z = 3x^2 + 3y^2 - 16$.