

HOMEWORK PROBLEM SET 5: DUE MARCH 10, 2017

110.302 DIFFERENTIAL EQUATIONS
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Question 1. Use Euler's formula to do the following:

- (a) Write $e^{i\pi}$ in the form $a + ib$.
- (b) Write 2^{2-2i} in the form $a + ib$.
- (c) Write π^{2i-1} in the form $a + ib$.
- (d) Show that

$$\cos t = \frac{e^{it} + e^{-it}}{2}, \quad \text{and} \quad \sin t = \frac{e^{it} - e^{-it}}{2i}.$$

Question 2. Solve the following:

- (a) $y'' - 6y' + 9y = 0$, $y(0) = 0$, $y'(0) = 2$.
- (b) $9y'' + 9y' - 4y = 0$.
- (c) $y'' + 2y' + 6y = 0$, $y(0) = 2$, $y'(0) = 4$.
- (d) $2y'' + 2y' + y = 0$.
- (e) $y'' + 2y' + 2y = 0$.

Question 3. For the ODE $ay'' + by' + cy = 0$, suppose $b^2 - 4ac < 0$, so that the two solutions to the characteristic equation for the ODE are $\lambda \pm i\mu$. Do the following:

- (a) Show that $u(t) = e^{\lambda t} \cos \mu t$ and $v(t) = e^{\lambda t} \sin \mu t$ are, in fact, solutions to the ODE.
- (b) Show that the Wronskian $W(u(t), v(t)) = \mu e^{2\lambda t}$, thereby establishing the linear independence of $u(t)$ and $v(t)$ on all of $\mathbb{R}$.
- (c) Do the same for the ODE in the case that $b^2 = 4ac$ by showing that $u(t) = e^{-\frac{b}{2a}t}$ and $v(t) = te^{-\frac{b}{2a}t}$ each solve the ODE and are linearly independent of each other as functions.

Question 4. Determine the value of $\alpha \in \mathbb{R}$ for which the solution to the IVP

$$y'' = \frac{-4y - 12y'}{9}, \quad y(0) = \alpha > 0, \quad y'(0) = -1,$$

separates solutions from being negative from those that are always positive.

Question 5. Given the following ODEs and the one solution given, write out a full fundamental set of solutions:

- (a) $t^2 y'' + 2ty' = 2y$, $t > 0$, $y_1(t) = t$.
- (b) $(x-1)y'' - xy' + y = 0$, $x > 1$, $y_1(x) = e^x$.