

HOMEWORK PROBLEM SET 4: DUE MARCH 3, 2017

110.302 DIFFERENTIAL EQUATIONS
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Question 1. Solve the following:

(a) $2y'' + 5y' + 2y = 0$, $y(0) = -1$, $y'(0) = 5$.

(b) $y'' - 5y' + 5y = 0$.

(c) $y'' = \frac{1}{6}(y + y')$.

(d) $4y'' = 3y'$, $y(0) = \frac{5}{2}$, $y'(0) = -2$.

Question 2. Construct a second-order, linear, homogeneous, IVP with constant coefficients whose particular solution is $y(t) = 4e^{3t} - e^{-2t}$.

Question 3. Solve the IVP $32y'' - 2y = 0$, $y(0) = 4$, $y'(0) = \alpha$, and find the unique value of $\alpha \in \mathbb{R}$ so that $\lim_{t \rightarrow \infty} y(t) = 0$.

Question 4. Find the maximum value of a function $y(t)$ that satisfies the following: (1) its value at $t = 0$ is 3, (2) its derivative at $t = 0$ is $-\frac{2}{3}$, and (3) the function is the difference of four times its first derivative and three times its second derivative.

Question 5. Given the ODE $\ddot{x} - (2\alpha - 1)\dot{x} + \alpha(\alpha - 1)x = 0$, where $\alpha \in \mathbb{R}$ is a constant, find the value(s) of α , if any, where all solutions tend to 0 as $t \rightarrow \infty$. Then find all value(s) of α , if any, where all non-zero solutions are unbounded as $t \rightarrow \infty$.

Question 6. Calculate the Wronskian of the following pairs of functions and determine the domain of the Wronskian function $W(f, g)(x)$:

(a) $f(x) = xe^{r_1x}$, $g(x) = xe^{r_2x}$.

(b) $f(x) = \cos^2 x$, $g(x) = 1 + \cos 2x$.

(c) $f(x) = x^2 + 1$, $g(x) = 2x$.

Question 7. Verify that $y_1(t) = 1$ and $y_2(t) = \sqrt{t}$ both solve the ODE $yy'' + (y')^2 = 0$, for $t > 0$, but $y(t) = c_1 + c_2\sqrt{t}$ is not a general solution to the ODE. Explain why this result does not contradict Theorem 3.2.2 in the text on Superposition.

Question 8. Determine the Wronskian of any two solutions to the ODE $t^2y'' - t(t+2)y' + (t+2)y = 0$ without actually solving the ODE.

Question 9. If the Wronskian of two functions $f(x)$ and $g(x)$ is t^2e^{2t} , and $g(t) = t$, then what is $f(x)$?

Question 10. For the ODE $ay'' + by' + cy = 0$, suppose that $a > 0$. Find conditions on a , b , and c so that the two solutions of the characteristic equation are real, distinct and negative.