

HOMWORK PROBLEM SET 2: DUE FEBRUARY 17, 2017

110.302 DIFFERENTIAL EQUATIONS
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Question 1. Solve the following separable differential equations by separating the variables

(a) $xy' - y = 2x^2y$, for $x > 0$. (Note: This ODE is also linear.)

(b) $yy' = -2t(1 + y^2)$, $y(0) = 1$.

Question 2. Determine where the solution to the IVP $y' = xy^2 + 2y^2$, $y(0) = 1$ attains its minimum value.

Question 3. Do problem 2.2.30 in the text (the 10th edition).

Question 4. Without solving the IVPs, determine the largest interval in which the solution is guaranteed to exist.

(a) $x(x - 4)y' + y = 0$, $y(2) = 1$.

(b) $(\ln t)\dot{z} + z - \cot t = 0$, $z(2) = 3$.

Question 5. State where in the ty -plane the solutions to the following ODEs are guaranteed to exist. Then also state where solutions will also be uniquely defined.

(a) $y'(1 - t^2 + y^2) = \ln |ty|$.

(b) $y' = \frac{y - t}{5y + 2t}$.

Question 6. Solve the IVP $z' = -z^3$, for $z(0) = z_0$ and determine how the interval in which the solutions exists depends on the initial value z_0 .

Question 7. Do problem 2.4.23 in the text (the 10th edition).

Question 8. For the following autonomous ODEs in the format $y' = f(y)$, do the following: (1) sketch the graph of $f(y)$ verses y , (2) determine the critical points (the places where equilibrium solutions exist) and classify each equilibrium as asymptotically stable, semi-stable, or unstable, (4) draw a phase line, and (5) draw enough trajectories in the ty -plane to completely exhibit solution behavior.

(a) $y' = y(y + 1)(y - 2)$.

(b) $y' = a\sqrt{y} - by$, where $a > 0$, $b > 0$, and $y \geq 0$.

(c) $y' = y^2(1 - y^2)$.

(d) $y' = y^2(4 - y)^2$.

Question 9. Suppose y_1 is a critical point of the ODE $\frac{dy}{dt} = f(y)$. Show that the equilibrium solution $y(t) \equiv y_1$ is asymptotically stable if $f'(y_1) < 0$ and unstable if $f'(y_1) > 0$.