HOMEWORK PROBLEM SET 12: DUE MAY 5, 2017

110.302 DIFFERENTIAL EQUATIONS
PROFESSOR RICHARD BROWN

Question 1. For the given first-order IVPs, do the following: (1) Approximate the solution at $t = 2$ by using Euler's Method with a step size of $h = .5$, (2) solve the ODE and calculate the difference between your approximate solution and the actual solution.

(a) $y' = 3 + t - y, \quad y(0) = 1.$

(b) $y' = 2y - 3t, \quad y(0) = 1.$

Question 2. Suppose that $x(t)$ solves the ODE $\dot{x} = \sqrt{x + t}$. Use Euler's Method to approximate $x(4)$ knowing $x(1) = 3$. Use a step size of $h = .5$.

Question 3. For the following, find the function that transforms to the expression given:

(a) $F(s) = \frac{3}{s^2+4}$.

(b) $F(s) = \frac{2}{s^2+3s-4}$.

(c) $F(s) = \frac{4}{(s-1)^3}$.

Question 4. For the following, Use the Laplace Transform to solve the following IVPs:

(a) $y'' - y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = -1.$

(b) $y'' - 4y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 1.$

(c) $y^{(4)} - y = 0, \quad y(0) = y''(0) = 1, \quad y'(0) = y'''(0) = 0.$

Question 5. Find the Laplace transform of the function $y(t)$ that satisfies the IVP

$$y'' + y = \begin{cases} t & 0 \le t < 1 \\ 2 - t & 1 \le t < 2 \\ 0 & t \ge 2, \end{cases} \qquad y(0) = y'(0) = 0.$$

You do not need to find $y(t)$.

Question 6. Sketch $g(t)$ on the interval $t \ge 0$ and find its Laplace Transform:

(a) $g(t) = u_1(t) + 2u_2(t) - 6u_4(t)$.

(b) $g(t) = f(t-2)u_2(t)$, where $f(t) = t^2$.

(c) $g(t) = u_2(t)(t-3) - (t-2)u_3(t)$.

Question 7. Solve the IVPs: [This is optional. But give these a try!]

(a) $y'' + 3y' + 2y = h(t), \quad y(0) = y'(0) = 0, \quad h(t) = \begin{cases} 1 & 0 \le t < 10 \\ 0 & t \ge 10. \end{cases}$

(b) $y'' + 2y' + 2y = f(t), \quad y(0) = 0, \quad y'(0) = 1, \quad f(t) = \begin{cases} 1 & \pi \le t < 2\pi \\ 0 & 0 \le t < 10 \text{ and } t \ge 2\pi. \end{cases}$