HOMEWORK PROBLEM SET 1: DUE FEBRUARY 10, 2017

110.302 DIFFERENTIAL EQUATIONS
PROFESSOR RICHARD BROWN

Question 1. In each of the ODEs below, draw a direction field (you can use technology). Based on the direction field, determine and describe the behavior of solutions $y(t)$ as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, then describe the dependency.

(a) $y' = 4 - 3y$.

(b) $y' = 4y - 3$.

(c) $y' = -y(2 - y)$.

(d) $y' = y + 2 - t$.

Question 2. Do text problems 1.1.15-1.1.20 (this is a quick matching exercise to help develop your intuition).

Question 3. For the following, determine the order of the ODE and whether the ODE is linear or nonlinear.

(a) $\frac{d^4y}{dt^4} + \frac{d^3y}{dt^3} - \frac{d^2y}{dt^2}\frac{dy}{dt} + y = 1$.

(b) $\frac{dy}{dt} + y\sin^2 t = 0$.

(c) $\frac{d^3y}{dt^3} + \sin(t + y) = e^t$.

(d) $(\ln t)\frac{d^2y}{dt^2} + \frac{1}{t}\frac{dy}{dt} = t^2 y$.

Question 4. For the following, verify that the given functions are solutions to the ODE.

(a) $2t^2y'' + 3ty' - y = 0,\ t > 0; \quad y_1(t) = \sqrt{t}, \quad y_2(t) = t^{-1}$.

(b) $y'' + y = \sec t,\ 0 < t < \frac{\pi}{2}; \quad y = (\cos t)\ln\cos t + t\sin t$.

Question 5. For the following, determine the values of r for which the given differential equation has solutions of the form given.

(a) $2y'' - 12y' + 10y = 0; \quad y(t) = e^{rt}$.

(b) $t^2y'' + 2ty' - 6y = 0,\ t > 0; \quad y(t) = t^r$.

Question 6. Solve the following linear differential equations.

(a) $2y' + y = 3t^2$.

(b) $y' + 2y = te^{-2t}, \quad y(1) = 0.$

(c) $ty' + (t+1)y = t, \quad t > 0, \quad y(\ln 2) = 1.$

(d) $y' = -ay + b$, for a and b positive numbers. In this case, sketch solutions for several different initial conditions on one graph.

Question 7. Solve the IVP

$$ty' + (t+1)y = 2te^{-t}, \quad y(1) = a, \quad t > 0,$$

and locate the particular value of $a = a_0$ for which the transition from one type of behavior to another occurs. Describe the behavior of the solution corresponding to a_0.

Question 8. For the IVP,

$$y' + \frac{2}{3}y = 1 - \frac{t}{2}, \quad y(0) = y_0,$$

find the value of y_0 for which the solution touches, but does not cross, the t-axis.