Some Guidelines for Good Mathematical Writing

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President's Message

Communicating mathematics well is an important part of doing mathematics. Many of us know from writing papers or giving talks that communicating effectively not only serves our audience but also clarifies and structures our own thinking. There is an art and elegance to good writing that every writer should strive for. And writing, as a work of art, can bring a person great personal satisfaction.

Within the MAA, we value exposition and mathematical communication. In this column, I'm sharing the advice I give my students to help them write well. There are more extensive treatments (e.g., see Paul Halmos's *How to Write Mathematics*), but I wanted a shorter introduction. So I developed the guidelines below.

Basics

Know your audience. This is the most important consideration for writers. Put yourself in your reader's shoes. What background can we assume of the reader? What terminology should we define? What kind of "voice" do we want to project: casual or professional, serious or inviting, terse or loquacious?

If you are a student writing solutions for a homework set and your professor has not specified your audience, a good rule of thumb is to assume you are writing to another student in the course who has not yet done the assignment. Though you may assume that she has attended all the same lectures and has read the same textbook, it is standard courtesy to remind your readers of any relevant items that they have learned recently from the class or textbook, or things they should know but might have forgotten.

For instance, if the concept of a rational number was only recently learned in class, you might insert "Recall that a rational number can be expressed as a fraction" before saying "since x is rational, $x = m/n$ where m and n are integers."

Set an invitational tone. It is traditional to create an inviting atmosphere in one's mathematical writing. In effect, we invite readers to join us in our reasoning process by writing in the present tense, using the pronoun "we" instead of "I" (e.g., "we construct a tangent plane..."), and directing the reader with gentle commands (e.g., "let n be . . . ," "recall that . . . ," or "consider the set of . . .").

Use complete sentences. All mathematics should be written in sentences. Open any mathematics text and you'll see that this is true. Equations, even displayed ones, have punctuation that helps you see where it fits in the context of a larger sentence. Consider this piece of writing:

$$(x-2)^2 + (x-1)^2 = 5^2\ 5^2 = 25$$
$$(x-2)^2 = x^2 - 4x + 4 + x^2 - 2x + 1 = 25.$$
$$2x^2 - 6x - 20$$
$$2(x+2)(x-5)\ x = \text{-}2, 5\ x > 0\ x = 5$$

Can you figure out what the writer is doing? What's being assumed? What's being proved? Where does one thought end and another begin? What's the relationship between these phrases? Some phrases are dangling, and others, as statements, are not even true. The reader should not have to figure out what the writer was thinking.

Now consider the work of another writer attempting the same problem:

Problem. Find a point on the line $y = x$ that is distance 5 from the point (2,1) and whose x-coordinate is positive.

Solution. The desired point is (5,5). To see this, we solve $(x-2)^2 + (x-1)^2 = 5^2$, an equation obtained from the distance formula in the plane. A little algebra turns this equation into:

$$2x^2 - 6x - 20 = 0.$$

Factoring the left side, we obtain

$$2(x+2)(x-5) = 0,$$

whose solutions are $x = -2$ and $x = 5$. Since we assumed $x > 0$, we have (5,5) as the desired point on the line $y = x$.

Here, the writer has clearly stated the problem and described her path to a solution. She has set an invitational tone, and every thought is expressed in a complete sentence. Now it is clear that x > 0 is a condition, not a result. Notice the punctuation in equations: one ended with a period because her thought was complete, the other ended with a comma because she wanted to continue the thought.

Since she assumed her audience could do algebra, she didn't bore them with algebraic manipulation, which would obscure the thread of her arguments. But she did show the crucial and most interesting piece: the factoring and its result. And she made sure she answered the original question.

Use words to give context to equations. Consider the difference in meaning between these three statements: "Let $A = 5$." "Suppose $A = 5$." "Therefore $A = 5$." Then reflect on the ambiguity of the statement "$A = 5$."

Avoid shorthand in formal writing. The many types of mathematical writing can be loosely grouped into formal and informal writing. Informal writing includes writing on a blackboard during lecture, or explaining something to a friend on a piece of scratch paper. Formal writing includes the kind of writing expected on a homework assignment or in a paper. There are differences in what is acceptable. For instance, in informal writing, it is common to use shorthand for quantifiers and implications: symbols such as $\forall$, $\exists$, $\Rightarrow$, $\Leftrightarrow$, or abbreviations such as "iff" and "s.t."

However, in formal writing, such shorthand should generally be avoided. You should write out "for all," "there exists," "implies," "if and only if," and "such that." Most other symbols are acceptable in formal writing, after defining them where needed. The membership symbol $\in$ is traditionally acceptable in formal writing, as are relations (e.g., $<$, $+$, $\cup$, etc.), variable names (e.g., x, y, z), and symbols for sets (e.g., $\mathbb{R}$). Here is an acceptable use of symbols in formal mathematical writing:

> Let A and B be two subsets of $\mathbb{R}$. We say A *dominates* B if for every $x \in A$ there exists $y \in B$ such that $y > x$.

Learn the etiquette. The above example also illustrates two common conventions of mathematical etiquette. It is customary to avoid beginning sentences or phrases with a number or symbol because that can be confusing. It is also customary to emphasize unfamiliar words that we are about to define, such as by italicizing them. Other etiquette can be learned by observing the norms used in your area of study.

Toward Elegance

Decide what's important to say. Writing well does not necessarily mean writing more. If your solution is too wordy, it can sometimes obscure the points you are making.

A well-written solution will present just enough details and highlight the most interesting or unexpected parts of the argument. What theorems or axioms were crucial in getting your solution, and where were they used? Your role as a writer is not primarily to give details (though that can be important). Your primary role is to give insight.

Highlight structure. If your argument is going to be a long one, with lots of technical details, then try to help the reader by summarizing the outline of your argument at the beginning. Then, throughout your writing, help your reader see how you are progressing through your outline.