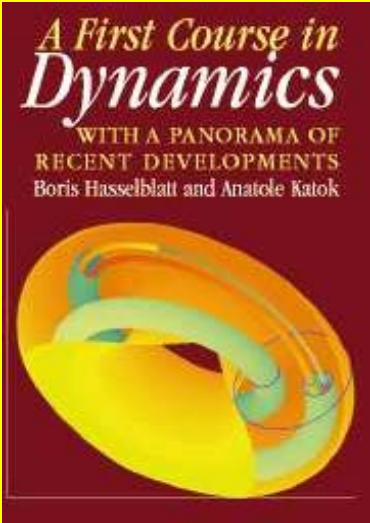


Math 110.421: Dynamical Systems



Spring 2015 Course Page

Instructor: [Professor Richard Brown](#)
Lecture: TTh 3:00pm - 4:15pm Room: Krieger 309
Teaching Assistant/Grader: [Xiyuan Wang](#)

Text:	A First Course in Dynamics , 1st Edition	Errata to the book.
	Hasselblatt, Boris and Katok, Anatole Cambridge University Press, 2003, ISBN 0 521 58750 6	

[Course Syllabus and Homework Assignment Schedule](#)

This is a first course in the study of the mathematics underlying the idea of what it means to model something mathematically. When we attempt to study a phenomenon mathematically, we seek a functional relationship between an unknown, measurable quantity, and one we do know and can control. This functional relationship means that we can use the known quantity to calculate values of the unknown quantity, predict its values, and study the properties of the dependence. Obviously, this involves the calculus of functions of one or more variables. Since vector calculus is involved, it also involves linear algebra. But in a deeper sense, dynamical systems involves the use of tools and concepts from all branches of mathematics, from topology, geometry and analysis, to algebra. The basic models in this study are either discrete, functions from a topological space to itself, or continuous, involving differential equation dictating flows on topological spaces. The spaces in general can be any set with a topology on it (a well-defined notion of open subset, or neighborhood), but will almost always be subsets of Euclidean Space in the course. We will use applications at times to study interesting behavior, but in general we will address a different question: Just what kind of dynamical information can be gleaned from a particular type of function or ODE on a particular type of space? We will start with very basic models exhibiting what we call simple dynamics, where almost all of the information can be described in very few words, and continue to the very complicated systems, where fully describing the dynamical behavior is complex. Along the way we will develop a language to describe the dynamical systems and their behavior, classify systems according to their properties, broaden our perceptions of what it means to "move around" in a space, and gain insight as to just how important the subject can be to understanding both the properties of functions and spaces as well as applications to the natural sciences and engineering.

For full disclosure, the above text will be required for the course. However, I am under contract with Oxford University Press to write my own text on this subject, and will be using my notes rather expensively in the course as I prepare a more final draft this semester. The entirety of these notes are available to you and your help in this endeavor will be an aspect of this course. I thank you in advance for this help and will happily acknowledge this help explicitly in the text.

For now, keep an active link to this page and site. I will update often with new information and any links I find relevant and/or useful. I will also place here announcements and such to keep you informed of anything I would like to know.

Welcome to the course! I love teaching this stuff, and hope you enjoy the time we have together.

Ordinary Differential Equations (specifically first-order, autonomous systems) will play an important role in this course, as the basic models for what we call continuous dynamical systems. Solving such things is often difficult and often impossible. Hence the necessity of a more qualitative approach which is the basis of the analysis we will conduct in this class. There are Java Applets, constructed long ago, that are quite useful in understanding the nature of Ordinary Differential Equations like this via an approximation of solutions. You can find some here:

[JAVA Applets for ODEs](#)

For now, pay attention to the Slope Field Calculators, particularly this [framed version](#), which will load in a separate window. I suggest you play with these applets.

Here is the main copy of [the Notes](#). I realized that the exercises did not compile correctly, so I unfroze and then refroze them. I will be grabbing many homework problems from here and the structure of the lectures will follow these notes closely.

The course thread:

- [Week 1, Lecture 1](#): These are taken directly from when I taught the course in 2013. But they are a good start. For now, please give these a read. They are little more than an introduction to just what a dynamical system is and some basic examples.
- Week 2, Lecture 1 (Notes: 5-17) will continue last weeks ideas, which you can see in the notes from then. I may start on the idea of linear as a beginning example, though.
- Week 2, Lecture 2: (Notes: 17-27) Today, we finish up Section 2.1 and move into 2.2 on the Contraction Principle. This principle, both in the real-line version as well as the more general versions, provide a good, though somewhat restrictive idea of simple dynamics in a metric space. Using the metric explicitly allows us to follow orbits of points by measuring their distances from each other and from fixed points easily. Contractions have the special property that distances tend to decay exponentially. A good property when it occurs. And the local version will form the basic idea with which we can move on from.
- Week 3, Lecture 1: (Notes: 27-36) Starting from the multidimensional version of the Contraction Principle, we notice that the structure of the theorem is the same as the one-dimensional version and entirely based on the metric. However, to understand how the derivative of a function in more than one variable may help to define the contractive constant, we look for ways to assign a number to the derivative matrix. One way was through the matrix norm defined by the Euclidean metric and show that if that is bounded by a constant less than one, the function is a contraction. Also, for a local version, if, in the neighborhood of a fixed point of a map, all of the eigenvalues of the derivative matrix at the fixed point are all of modulus less than one, map is a contraction locally. This is the basic idea behind the Newton Raphson Method. We also mentioned other applications like Picard Iterations and Heron's Method. Then we moved into interval maps and defined cobwebbing at the end as a means to "see" where orbits go under iteration of a map on an interval.
- Week 3, Lecture 2: (Notes: 36-41)
- Week 4, Lecture 1: (Notes: 42-47)
- Week 4, Lecture 2: (Notes: 47-52)
- Week 5, Lecture 1: (Notes: 52-59)
- Week 5, Lecture 2: (Notes: 60-69)
- Week 6, Lecture 1: (Notes: 69-77)
- Week 6, Lecture 2: (Notes: 78-94)
- Week 7, Lecture 1: (Notes: 94-106)
- Week 7, Lecture 2: (Notes: 106-113)
- Week 8, Lectures 1 and 2: (Notes: 113 - 134)
- Week 9, Lectures 1 and 2: (Notes: 134 - 148)
- Week 10, Lectures 1 and 2: (Notes: 148 - 161)
- Week 11, Lectures 1 and 2: (Notes: 161 - 175)

Homework Sets:

- **Problem Set 1:** (Due February 11) Exercises 3,8,11,12,15,19,21,25,28,30,33,36 in [the Notes](#). Plus the following: On a separate sheet of paper, marked with your name and date, prove the following: The area of a triangle in the plane is half the length of its base times its height.
- **Problem Set 2:** (Due February 18) Exercises 37,38,39,51,53,55,56,57,59,60,62 in [the Notes](#).
- **Problem Set 3:** (Due February 25) Exercises 63,64,65,67,70,71,73 in [the Notes](#).
- **Problem Set 4:** (Due March 3) Exercises 74,76,78,83,89,90 (forget the contraction part),91,92,95 in [the Notes](#).
- **Problem Set 5:** (Due March 10) Exercises 96,97,99,100,105-9,115,117-9 in [the Notes](#).
- **Problem Set 6:** (Due March 24) Exercises 121-3,126-8 in [the Notes](#), and EP25 and EP27 below the schedule in the syllabus page.
- **Problem Set 7:** (Due March 31) Exercises 129,130,132,133,134,137,138,139,142,144 in [the Notes](#).
- **Problem Set 8:** (Due April 7) Exercises 146,147,152,153,154,156,157,158 in [the Notes](#).
- **Problem Set 9:** (Due April 14) Exercises 159-164,168,169 in [the Notes](#).
- **Problem Set 10:** (Due April 21) Exercises 170-174,181,182,184,185 in [the Notes](#).

The Final Problem set

- No Problem Set 11.
- The [final problem set](#) is due On Wednesday, May 11, 3pm.

Mandelbrot and Julia Sets

- <http://tilde.club/~david/m/#>
- <http://www.sinistercircuits.com/julia/live/index.html>
- http://www.daviddarling.info/images/Mandelbrot_set.jpg
- <http://www.emba.uvm.edu/~jxyang/teaching/Math266notes9.pdf>

Research Questions relevant to this course which you are now at a level to understand and contribute. The following are not exercises, as in I know the answer. These are real undergraduate research projects.

1. **Piecewise contractions:** A differentiable map on $[0,1]$ whose derivative is strictly bounded in absolute value above by 1 is a contraction. To what extent can we say the same for a map on $[0,1]$ which is differentiable everywhere except for a single jump discontinuity? It is easy to construct such a map that has no fixed point. But must it have a single asymptotically stable periodic orbit. Such a map we will call a piecewise contraction.
2. **Increasing maps on intervals and autonomous ODEs.** An increasing map on $[0,1]$ can be studied by producing a "phase line" in a manner similar to that of a first order autonomous ODE. In fact, it seems like we can always construct an increasing function whose dynamics are the same as that of a time-1 map of an autonomous ODE. What is the relationship between increasing maps and autonomous ODEs?

For those of you who would like help outside of that of the professor or TAs, there is a free service offered by the [Mathematics Help Room](#). Click for more details.

last updated 01/14/2016