FINAL PROBLEM SET

DYNAMICAL SYSTEMS (110.421)
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1. General Information

This homework set is to be considered the final examination for the course, and is **due on Friday, May 14, 2010 at noon**. You may drop off your solutions to this take home final examination any time before this due date. You may also scan and email me your solutions. Good luck.

2. Problems

Exercise 2.1. Show that a contraction mapping on a metric space cannot have more than one fixed point.

Exercise 2.2. Graph on the unit square in $\mathbb{R}^2$ a homeomorphism f of the unit circle S^1, with exactly two fixed points and where $f(0) = \frac{1}{2}$. Using the graph, classify the fixed points in terms of their stability (and justify your classification).

Exercise 2.3. For general $n \in \mathbb{Z}$, lift the map $f : S^1 \to S^1$, $f(z) = z_0 z^n$, $|z_0| = 1$ (using the unit modulus complex numbers to denote points on S^1) to a map on $\mathbb{R}$, and verify that it is indeed a lift.

Exercise 2.4. Show that for a translation on the two torus $\mathbb{T}^2$, every point is recurrent.

Exercise 2.5. Find the fixed, period-2 and period-3 points for the hyperbolic toral map given by the matrix $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$.

Exercise 2.6 (5.1.2 in text). Suppose $n, m \in \mathbb{Z}$ are such that 1, $\sqrt{m}$, and $\sqrt{n}$ are rationally dependent. What does this imply about n and m?

Exercise 2.7 (7.2.7 in text). Consider a 2×2 integer matrix L without eigenvalues of absolute value 1 and with $|\det L| > 1$. Prove that the induced noninvertible hyperbolic linear map $F_L : \mathbb{T}^2 \to \mathbb{T}^2$ is topologically mixing.

Exercise 2.8 (7.4.1 in text). Prove that for $\lambda \geq 1$, every bounded orbit of the quadratic map f_λ (on $\mathbb{R}$) is in $[0, 1]$.

Exercise 2.9. Find an attracting, periodic point of minimal period 6 for the logistic map $f(x) = \lambda x(1-x)$. State the parameter value for this point, and approximate the size of the interval of parameter values for which this periodic orbit exists (Hint: you are looking for two bifurcation values for λ here.)

Exercise 2.10. There is an assertion on page 243 of the text: Let C_α be the Canter subset of the unit interval constructed by deleting the middle interval of relative length $1 - \frac{2}{\alpha}$ at each stage. Then

$$bdim(C_\alpha) = \frac{\log 2}{\log \alpha}.$$

Prove this assertion. What is the box dimension of the Cantor set constructed when one removes the middle fourth of the remaining intervals at each stage?

Exercise 2.11. For $I = [0, 1]$, let the logistic map $f_\lambda : I \to I$ have parameter value $\lambda \in [2, 3]$. Show f_λ^2 is a contraction when restricted to the interval $\left[1 - x_\lambda, f\left(\frac{1}{2}\right)\right]$.